

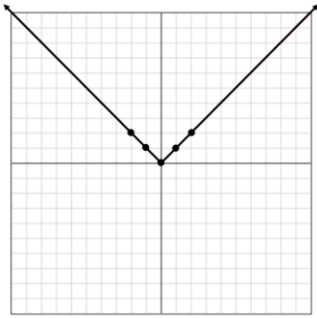
**Part I: Graphing Parent Functions and the Rule of Four**

***Quick Review – The Rule of Four***

There are four basic ways of representing functions: **verbal, algebraic, numerical, and graphical.**

- Verbal includes listing the name of the function and describing its characteristics.
- Algebraic includes writing the equation of the parent function.
- Numerical includes writing 5 key anchor points and features, such as the location of any hole(s) and the equations for all asymptotes (if any). Choose “nice”  $x$  values – often we use  $x$  values of  $-2, -1, 0, 1, 2$  unless the function is not defined for those values or those values are hard to compute.
- Graphical includes plotting points and sketching the graph.

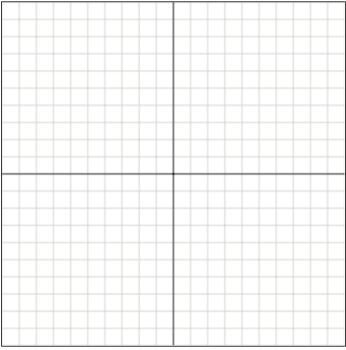
**Example: Absolute Value Parent Function**

<p><b>Verbal:</b> Name: <i>Absolute Value Function</i></p> <p>Domain: <math>(-\infty, \infty)</math> Range: <math>[0, \infty)</math></p>	<p><b>Algebraic (Formula):</b></p> $y =  x  = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$
<p><b>Numerical (Anchor Points):</b></p> <p><math>(-2, 2)</math> <math>(-1, 1)</math> <math>(0, 0)</math> <math>(1, 1)</math> <math>(2, 2)</math></p>	<p><b>Graphical:</b></p> 

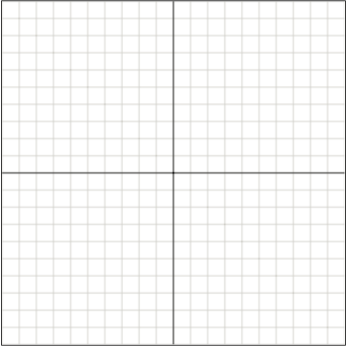
**Problem Set I: Graphing Parent Functions and The Rule of Four**

Given the name of the following parent functions, complete the rule of four charts for each.

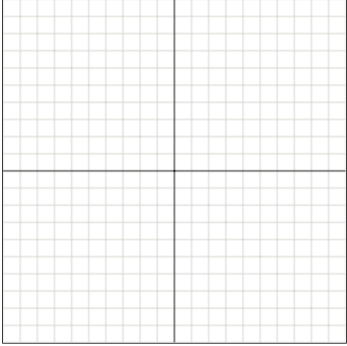
1. Quadratic Function

<b>Verbal:</b> Name: <i>Quadratic Function</i> Domain:                      Range:	<b>Algebraic (Formula):</b>
<b>Numerical (Anchor Points):</b>	<b>Graphical:</b> 

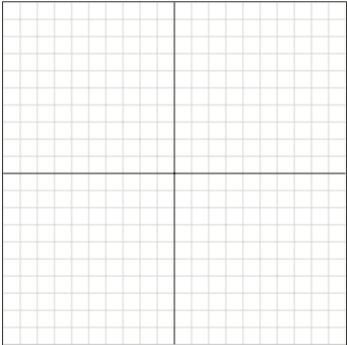
2. Square Root Function

<b>Verbal:</b> Name: <i>Square Root Function</i> Domain:                      Range:	<b>Algebraic (Formula):</b>
<b>Numerical (Anchor Points):</b>	<b>Graphical:</b> 

3. Cubic Function

<b>Verbal:</b> Name: <i>Cubic Function</i> Domain:                      Range:	<b>Algebraic (Formula):</b>
<b>Numerical (Anchor Points):</b>	<b>Graphical:</b> 

4. Cube Root Function

<b>Verbal:</b> Name: <i>Cube Root Function</i> Domain:                      Range:	<b>Algebraic (Formula):</b>
<b>Numerical (Anchor Points):</b>	<b>Graphical:</b> 

5. Reciprocal Function

<b>Verbal:</b>	<b>Algebraic (Formula):</b>
Name: <i>Reciprocal Function</i>	$y = \frac{1}{x}$
Domain:                      Range:	
<b>Numerical (Anchor Points):</b>	<b>Graphical:</b>

6. Reciprocal Square Function

<b>Verbal:</b>	<b>Algebraic (Formula):</b>
Name: <i>Reciprocal Square Function</i>	$y = \frac{1}{x^2}$
Domain:                      Range:	
<b>Numerical (Anchor Points):</b>	<b>Graphical:</b>

## Part II: Identifying Characteristics of Functions

### Quick Review – Proper Notation

When we refer to a portion of a function, we describe the x-values for that portion using either Set or Interval Notation:

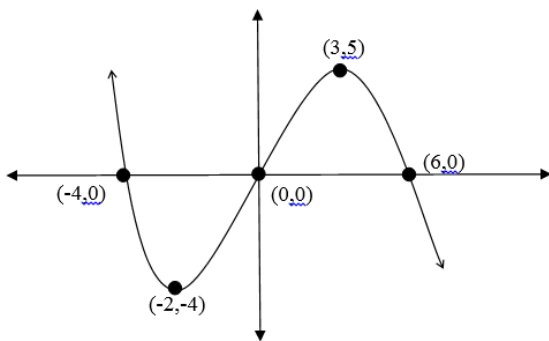
**Set Notation:**  $\{x|x \geq 3\}$  means set of all x values such that x is greater than or equal to 3

**Interval Notation:**  $[3, \infty)$  means all values of x from 3 to infinity including 3

The brackets and parentheses with Interval Notation are very important, for they tell us whether or not the endpoints of the interval are included in the set.

- ] and [ mean that the endpoint is included and that the function is defined at that point.
- ) and ( mean that the endpoint is not included or that the function is not defined at that point -- always use a parenthesis with an infinity symbol
- An interval can be closed on one end and open on the other.
- $\cup$  is a union symbol and means to include all values from the first interval and all values from the second interval.
- $\cap$  is an intersection symbol and means to include only the values that are in **both** the first interval and the second interval.

### Example: Identifying Characteristics of a Function



#### Characteristics

**Increasing:**  $[-2, 3]$

**Decreasing:**  $(-\infty, -2] \cup [3, \infty)$

**Constant:** Never

**Global/Absolute Maximum:** None

**Global/Absolute Minimum:** None

**Local/Relative Maximum:** point  $(3, 5)$

A local max value of 5 occurs at  $x = 3$ .

**Local/Relative Minimum:** point  $(-2, -4)$

A local min value of -4 occurs at  $x = -2$ .

**Positive:**  $(-\infty, -4) \cup (0, 6)$

**Negative:**  $(-4, 0) \cup (6, \infty)$

**Zero:**  $\{x|x = -4, 0, 6\}$

**Concave Up:**  $(-\infty, 0)$

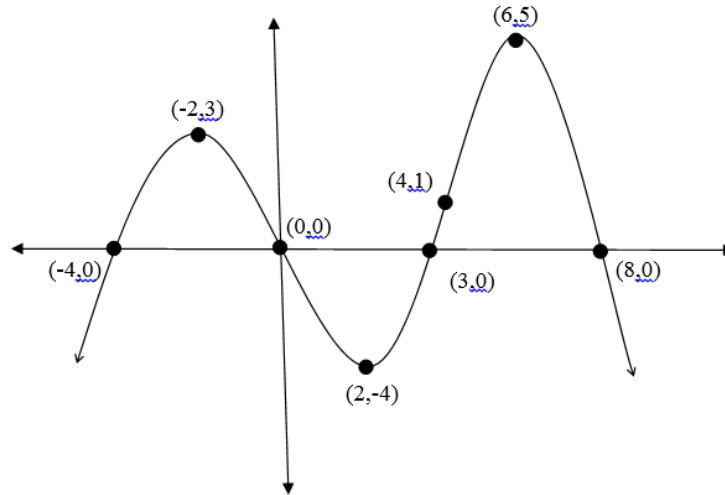
**Concave Down:**  $(0, \infty)$

**Points of Inflection:**  $(0, 0)$

**End Behavior:**  $f(x) \rightarrow -\infty$  as  $x \rightarrow \infty$

$f(x) \rightarrow \infty$  as  $x \rightarrow -\infty$

**Problem Set II: Identifying Characteristics of Functions**



Fill in the characteristics for the function above using proper notation. Assume  $(0, 0)$  and  $(4, 1)$  are the points of inflections.

Characteristics
Increasing:
Decreasing:
Constant:
Global/Absolute Maximum:
Global/Absolute Minimum:
Local/Relative Maximum:
Local/Relative Minimum:
Positive:
Negative:
Zero:
Concave Up:
Concave Down:
Points of Inflection: $(0,0)$ and $(4,1)$
End Behavior:

### **Part III: Solving Equations and Inequalities**

Students should be able to solve linear, quadratic, polynomial, rational, radical, and absolute value equations and inequalities.

**Problem Set III-A:** Solve the following equations. You should NOT use a calculator to solve. All solutions should be given as an exact answer (preferably simplified) – NO decimal approximations.

1.  $[6 - 4x + 2(x - 7)] - 52 - 3(2x - 4) = 6[3(2x - 3) + 6]$

2.  $8 - 3\left|\frac{1}{2}b - 4\right| = 2$

3.  $-2(2x - 3)^2 + 14 = 0$

4.  $4t^3 + 4t^2 - 2t = 0$

5.  $\frac{x}{x+2} - 4 = \frac{x+1}{x}$

6. 
$$\frac{2x}{x-1} + \frac{1}{x-3} = \frac{2}{x^2 - 4x + 3}$$

7. 
$$2\sqrt{4-y} + 10 = 12$$

8. 
$$\sqrt{x+2} = 6 - \sqrt{7x+2}$$

9. 
$$x^{2/3} + 3x^{1/3} + 2 = 0$$

10. 
$$p(2p-5)^2 - 3(2p-5) = 0$$



**Problem Set III-B:** Solve the following inequalities. Show all work including any sign chart analysis.

11.  $-6 \leq 1 - 4(x + 2) \leq 16$

12.  $|1 - 2x| < 4$

13.  $|2 - 5x| > 0$

14.  $6x^2 - 7x < 20$

15.  $\frac{3}{x-2} - \frac{1}{x-4} \leq 0$

## Part IV: Function Transformations

### **Quick Review – Graphing with the Transformation Approach**

There are three types of transformations:

- horizontal/vertical **reflections** (flip over x-axis or y-axis)
- horizontal/vertical **scaling** (stretching and shrinking)
- horizontal/vertical **translations** (shifts up/down or left/right).

Transformation Rule  $(x_o, y_o) \rightarrow (x_n, y_n)$  describes how to take a parent anchor point and transform it to the corresponding “sibling” anchor point.

### **Example: Graphing a Transformed Equation and Writing a Transformation Rule**

Graph:  $y = \frac{1}{3}(-2x + 8)^3 - 5$

- a. Decide what **parent** function is in the equation,  $y = \frac{1}{3}(-2x + 8)^3 - 5$ .

Parent Function:  $y = x^3$

- b. Find the **horizontal** transformations by solving the expression *inside* the parent function for  $x_n$ .

$$y = \frac{1}{3}(-2x + 8)^3 - 5 \longrightarrow x_o = -2x_n + 8$$
$$x_n = -\frac{1}{2}x_o + 4$$

Find the **vertical** transformations by looking at the *outside* of the parent function.

$$y = \frac{1}{3}(-2x + 8)^3 - 5 \longrightarrow y_n = \frac{1}{3}y_o - 5$$

Write the Transformation Rule:  $(x_o, y_o) \rightarrow \left(-\frac{1}{2}x_o + 4, \frac{1}{3}y_o - 5\right)$

- c. Verbally describe the transformations. You can determine these from the rule.

- Horizontal flip (reflection) over the y-axis
- Horizontal shrink by  $\frac{1}{2}$
- Horizontal Shift Right 4 units
- Vertical shrink by  $\frac{1}{3}$
- Vertical Shift Down 5 units

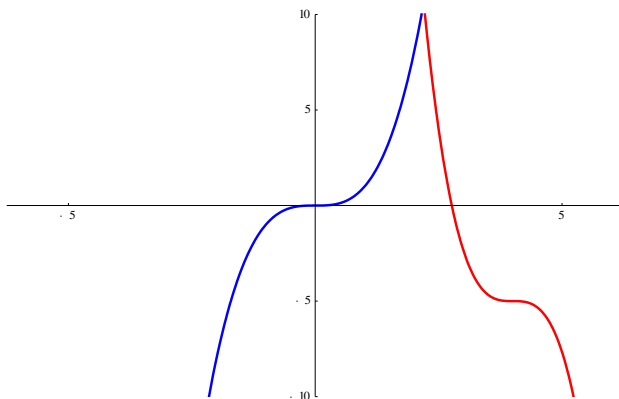
- d. Make a list of Anchor Points on the Parent Function (include any critical points on the graph, i.e. vertices, asymptotes, points of inflection). Take each point and substitute the  $x$  and  $y$  coordinates into the Anchor Point Rule:

$$(x_o, y_o) \rightarrow (x_n, y_n)$$

$x_o$	$y_o = (x_o)^3$	$-\frac{1}{2}x_o + 4$	$\frac{1}{3}y_o - 5$
-2	-8	5	-7.6
-1	-1	4.5	-5.3
0	0	4	-5
1	1	3.5	-4.6
2	8	3	-2.3

Notice the old point of inflection  $(0, 0)$  has moved to  $(4, -5)$ , the new point of inflection.

- e. Sketch both graphs and note how the parent graph has been changed. Use different colors for the two graphs. Show all the old and new anchor points):



### **Problem Set IV: Function Transformations**

Find the transformation rule given the transformed equation:

1.  $y = \sqrt{2x-3} + 1$   $(x_o, y_o) \rightarrow ( \text{_____} , \text{_____} )$   
parent \_\_\_\_\_

2.  $y = \frac{2}{3}(4x+1)^3 - 3$   $(x_o, y_o) \rightarrow ( \text{_____} , \text{_____} )$   
parent \_\_\_\_\_

Find the transformed equation given the transformation rule and parent:

3.  $(x_o, y_o) \rightarrow \left( x_n = -2x_o + 3, y_n = \frac{1}{2}y_o + 4 \right)$  with parent  $y = x^3$

4.  $(x_o, y_o) \rightarrow \left( x_n = \frac{2}{3}x_o - \frac{4}{9}, y_n = -3y_o - 2 \right)$  with parent  $y = \sqrt{x}$

5.  $(x_o, y_o) \rightarrow (x_n = 4x_o - 1, y_n = 3y_o + 2)$  with parent  $y = \frac{1}{x^2}$

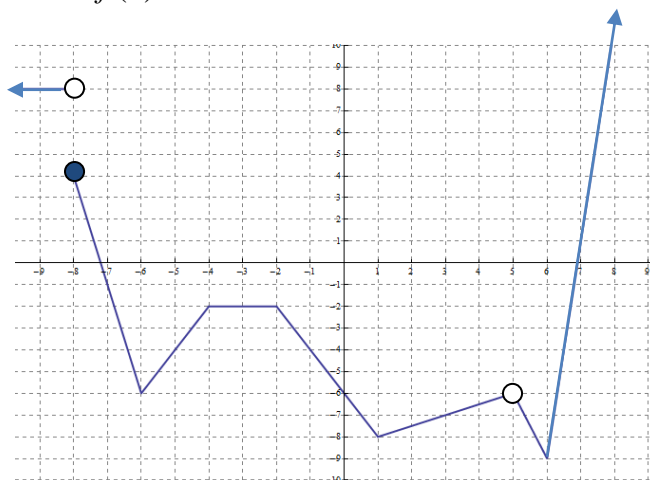
6. Graph the indicated transformation given the function  $f(x)$  shown below.

$$y = \frac{1}{2}f(3x-1) - 2$$

**Transformation rule**

$$(x_o, y_o) \rightarrow (x_n = \underline{\hspace{2cm}}, y_n = \underline{\hspace{2cm}})$$

**Numerical (Anchor Points):**



7. State the parent function and the transformation rule, describe the transformations verbally, find anchor points for both, and then graph both the parent and the transformed equation on the same plane.

$$y = -|2x - 1| + 3$$

Parent Function: \_\_\_\_\_

$(x_0, y_0) \rightarrow ( \text{_____} , \text{_____} )$

Verbal Description of Transformations:

