

For each of the following functions

1. Make a table of values for the function that clearly shows the main points. Please include **AT LEAST 6 points in your table**. These important points should include any key features such as:
 - The roots (x-intercepts)
 - Y-intercepts
 - Asymptotes : vertical and horizontal
 - Local maximum and minimum values.
 - Period if applicable

Your table should be big enough to illustrate these features

2. Sketch the graph for the function. When you sketch the graph be sure that all the points in your table are clearly shown on the graph. That is: if it is in the table, I want to see the POINT (a dot) on the graph. Also label the following:
 - Label by name any local maximum or minimum values eg. This is a **local max** and use an arrow to point to the dot on the graph.
 - Sketch asymptotes with a “broken line”.
 - Show in the table the roots and y-intercept

Here are your functions:

a) $y = \frac{x+2}{x^2-6x+5}$

b) $y = e^{3x} - 1$

c) $y = -e^{x^2}$

d) $y = \ln(x \cdot e)$

e) $y = 2\sin\left(\frac{x}{3} + \frac{\pi}{4}\right)$

Be sure to do this problem in RADIANS: Leave points in terms of pi

- II For each of the following, find the **area** described.
- Be sure to sketch the picture described. Draw the functions for the given domain
 - Shade the region enclosed by the functions described.
 - Show how you calculated the total area described.
- a) The area **enclosed by** the following lines:
- $y = 2x$
 - $y = 3$
 - $y = -2x + 8$
 - The horizontal x-axis ($y = 0$)
- b) The area enclosed by the functions
- $y = \sqrt{16 - x^2}$
 - The horizontal x-axis ($y = 0$)
- c) Use 2 rectangles to estimate the area enclosed by the graphs of:
- $y = 2^x$
 - $x = 0$
 - $x = 3$
 - The x-axis: $y = 0$
- Give an explanation for your estimate and tell me if the estimate is TOO HIGH or TOO LOW. (Hint: draw rectangles inside or outside of your picture and use a calculator to help you find the areas of the rectangles)

- III Find the “inverse function” for each of the following:
Show all work for FINDING the inverse function.
Graph the inverse function and the original function on the same axis (x-y plane)
You should also draw the line $y = x$ as a broken line. Recall that a function and its inverse are symmetrical about the line $y = x$.

- a) $y = 2x^2 - 1; \quad x \geq 0$
- b) $y = e^{2x}$
- c) $y = \cos x; \quad 0 \leq x \leq \pi$

IV Summation: Find the following sums and answer the related questions:

a)
$$\sum_{n=1}^{32} \frac{1}{n} =$$

What do you think will happen to the sum as n gets larger?

Hint: Consider the sum of the first 64 terms!

b)
$$\sum_{n=1}^{16} \frac{1}{n^2} =$$

What do you think will happen to the sum as n gets larger?

c)
$$\left(1 + \frac{1}{n}\right)^n =$$

What do you think will happen to the terms of this sequence as n gets larger?

Do this for the following values to try to get a “feel” for what is happening: Use these values of n: 1, 10, 100, 1000, 10 000, 100 000 This should give you an estimate for the number.

Do you recognize the number you see?

What number is this approaching as n approaches infinity?

NOTE ABOUT c Above:

Recall:

- As n gets very large, $\frac{1}{n} \rightarrow 0$
- As n gets very large, $(1.00001)^n \rightarrow \infty$ That means the expression get REALLY big for big values of “n”
- $(1)^n = 1$ for any value of n.
- Notice the above answer shows that some of the obvious rules may not apply when they are in conflict in the same problem.

V A. Find the following composition of functions:

Given the following:

$$f(x) = \sin x, \quad g(x) = 2x^2 - 3x + 1, \quad h(x) = \frac{5}{x+1}$$

Find:

a) $f(h(x)) =$

b) $g(h(x)) =$

c) $h(g(x)) =$

B. Write the following function as the composition of TWO functions:
(decompose into two functions)

Ex: $h(x) = \frac{3}{x^2 + 5x}$ is the composition:

$$h(x) = f(g(x)) \text{ where: } f(x) = \frac{3}{x}, \text{ and } g(x) = x^2 + 5x$$

Let $f(x)$ be the “big picture) and $g(x)$ be the details within

a) $h(x) = (x^2 + 5x + 6)^4$

b) $h(x) = \sin(x^2 + 6x + 1)$

c) $h(x) = \frac{6}{(3x + 5)^2}$

VI Add/Subtract or Multiply/Divide the following:

a) $\frac{3}{x+4} + \frac{5}{x-2} =$

b) $\frac{2}{(x-3)} + \frac{5}{x^2 - 6x + 9} =$

c) $\frac{2}{5} + \frac{1}{2} =$

d) $\frac{\frac{4}{3}x^{\frac{3}{2}}}{\frac{9}{2}} =$

e) $x^{\frac{1}{2}} \cdot x^{\frac{2}{3}} =$

f) $\sqrt{x} \cdot (x^2 + x^3 \sqrt{x})$

g) $e^x e^2 =$

h) Simplify as a single product or sum: $10^{k+3} =$

VII Solve for y:

a) $\ln y = x + 2$

b) $3xy + 5y = 2x + 7$

c) $3x^2y - 18 = 2xy + 5x^2$

VIII. Graphs/Properties of Functions: You may want to organize the following in a way you can save and use for a reference for the year. This section does not need to be IN THE COMPOSITION BOOK if you choose, but it can be. It must still be turned in.

Fill in the following table AND GRAPH each of the following

	The Function	Domain	Range	Type of Symmetry: x-axis, y-axis, origin	Odd, Even, Neither	Intervals of Increasing/Decreasing
1.	$y = 1$					
2.	$y = x$					
3.	$y = x^2$					
4.	$y = x^3$					
5.	$y = x^4$					
6.	$y = \sqrt{x}$					
7.	$y = \sqrt[3]{x}$					
8.	$y = \frac{1}{x}$					
9.	$y = \frac{1}{x^2}$					
10.	$y = x $					
11.	$y = \lfloor x \rfloor$					
12.	$y = \ln(x)$					
13.	$y = e^x$					
14.	$y = \sin x$					
15.	$y = \cos x$					
16.	$y = \tan x$					

The following Trigonometric Identities **MUST** be
memorized

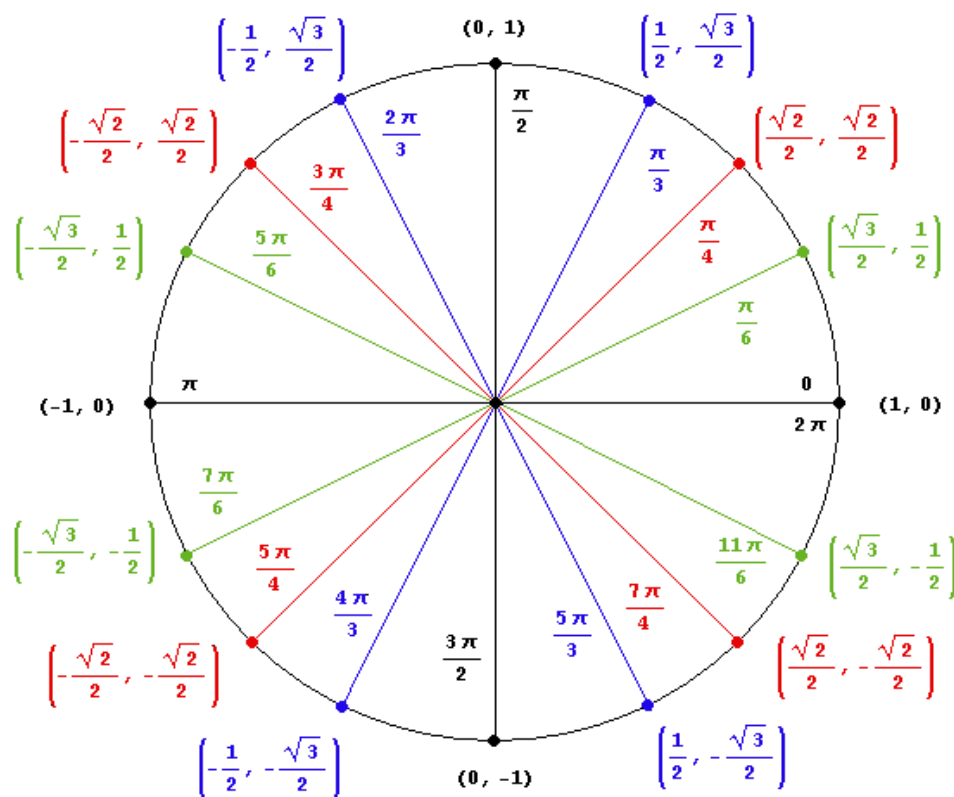
Reciprocal Identities	Quotient Identities	Pythagorean Identities											
$\sin x = \frac{1}{\csc x}$ $\csc x = \frac{1}{\sin x}$ $\cos x = \frac{1}{\sec x}$ $\sec x = \frac{1}{\cos x}$ $\tan x = \frac{1}{\cot x}$ $\cot x = \frac{1}{\tan x}$	$\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$	$\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$											
Co-Function Identities		Odd/Even Identities											
$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$ $\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$ $\csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta$ $\sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta$ $\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$ $\cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta$		<table style="width: 100%; border: none;"> <tr> <td style="text-align: center; width: 50%;"><u>Odd</u></td> <td style="text-align: center; width: 50%;"><u>Even</u></td> </tr> <tr> <td>$\sin(-\theta) = -\sin \theta$</td> <td>$\cos(-\theta) = \cos \theta$</td> </tr> <tr> <td>$\csc(-\theta) = -\csc \theta$</td> <td>$\sec(-\theta) = \sec \theta$</td> </tr> <tr> <td>$\tan(-\theta) = -\tan \theta$</td> <td></td> </tr> <tr> <td>$\cot(-\theta) = -\cot \theta$</td> <td></td> </tr> </table>		<u>Odd</u>	<u>Even</u>	$\sin(-\theta) = -\sin \theta$	$\cos(-\theta) = \cos \theta$	$\csc(-\theta) = -\csc \theta$	$\sec(-\theta) = \sec \theta$	$\tan(-\theta) = -\tan \theta$		$\cot(-\theta) = -\cot \theta$	
<u>Odd</u>	<u>Even</u>												
$\sin(-\theta) = -\sin \theta$	$\cos(-\theta) = \cos \theta$												
$\csc(-\theta) = -\csc \theta$	$\sec(-\theta) = \sec \theta$												
$\tan(-\theta) = -\tan \theta$													
$\cot(-\theta) = -\cot \theta$													

Double Angle Identities	Half Angle Identities
$\sin 2x = 2 \sin x \cos x$	$\sin^2 x = \frac{1 - \cos 2x}{2}$
$\cos 2x = \cos^2 x - \sin^2 x$	
$\cos 2x = 2 \cos^2 x - 1$	$\cos^2 x = \frac{1 + \cos 2x}{2}$
$\cos 2x = 1 - 2 \sin^2 x$	

The Radian Measures and Coordinates **MUST** be **memorized**

Remember: $\sin \theta = \frac{y}{r} = y - \text{coordinate}$, $\cos \theta = \frac{x}{r} = x - \text{coordinate}$, and

$$\tan \theta = \frac{y}{x} = \frac{y - \text{coordinate}}{x - \text{coordinate}}$$



KNOW AND UNDERSTAND ALL THE FOLLOWING FORMULAS and their uses.

Equation of a line:

$$y = mx + b$$

$$y - y_1 = m(x - x_1)$$

$$Ax + By = C$$

Slope of a line:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Parallel lines have the same slope

Perpendicular lines have slopes that are “negative reciprocals”

Area of a rectangle is length * width