Prerequisites for Calculus - 1.1 Linear Equations

This is a short review of linear equations (lines) and what we will need to know this year while we are beginning our journey into AP Calculus AB/BC. This set of notes will also help you with the use of your TI-83/84. Throughout these notes certain words and definitions will be boxed in which denoting their importance for the course. These words and definitions should be committed to memory.

> Definition: Net Change
> The distance an object moves from an initial (x_1, y_1) to a final (x_2, y_2) position measured both horizontally and vertically. The mathematical symbol for change or difference is the uppercase Greek letter delta (Δ).
>
> Horizontal Net Change: $\Delta x = x_2 - x_1$ Vertical Net Change: $\Delta y = y_2 - y_1$

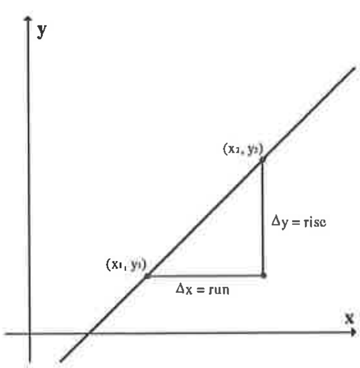
Example 1
Find the horizontal and vertical change of the coordinates $(3, 2)$ and $(5, 5)$.

$\Delta x = x_2 - x_1$ $\Delta y = y_2 - y_1$
$\Delta x = 5 - 3$ $\Delta y = 5 - 2$
$\Delta x = 2$ $\Delta y = 3$

Example 2
Find the horizontal and vertical change of the coordinates $(-1, 4)$ and $(2, -5)$.

$\Delta x = x_2 - x_1$ $\Delta y = y_2 - y_1$
$\Delta x = 2-(-1)$ $\Delta y = -5 - 4$
$\Delta x = 3$ $\Delta y = -9$

> Definition: Slope
> The relationship between the distances an object moves from an initial (x_1, y_1) to a final (x_2, y_2) position measured both horizontally and vertically.
>
> Slope: $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$



Example 3
Find the slope of the coordinates $(-5, -3)$ and $(6, 8)$.

$$m = \frac{\Delta y}{\Delta x} = \frac{8-(-3)}{6-(-5)}$$
$$m = \frac{\Delta y}{\Delta x} = \frac{11}{11} = 1$$

Example 4
Find the slope of the coordinates $(1, 3)$ and $(3, 7)$.

$$m = \frac{\Delta y}{\Delta x} = \frac{7-3}{3-1}$$
$$m = \frac{\Delta y}{\Delta x} = \frac{4}{2} = 2$$

The equation for slope can be manipulated to give us equations that represent the line which connects the two coordinates.

To receive the first linear equation from the slope formula we will begin by multiplying both sides by the denominator (cross multiplying).

$$m=\frac{y_2-y_1}{x_2-x_1} \quad \Rightarrow \quad (x_2-x_1)m=\frac{y_2-y_1}{x_2-x_1}(x_2-x_1) \quad \Rightarrow \quad y_2-y_1=m(x_2-x_1)$$

The equation is converted a little more by removing the subscripts from y_2 and x_2, thus allowing them to become variables. This type of linear equation is known as Point-Slope Form $y-y_1=m(x-x_1)$.

Example 5
Find the linear equation in Point-Slope Form which contain coordinates $(-5, -3)$ and $(6, 8)$.

$$m=\frac{\Delta y}{\Delta x}=\frac{11}{11}=1$$

$$y-8=1(x-6)$$

or

$$y+3=1(x+5)$$

Example 6
Find the linear equation in Point-Slope Form which contain coordinates $(1, 3)$ and $(3, 7)$.

$$m=\frac{\Delta y}{\Delta x}=\frac{4}{2}=2$$

$$y-7=2(x-3)$$

or

$$y-3=2(x-1)$$

Point-Slope Form may be refined more by isolating the y, distributing the m, and combing the like terms.

$$y-y_1=m(x-x_1) \quad \Rightarrow \quad y=mx-mx_1+y_1 \quad \Rightarrow \quad \text{Let } b=-mx_1+y_1 \quad \Rightarrow \quad y=mx+b$$

This type of linear equation is known as Slope-Intercept Form $y=mx+b$ (intercept means the y-intercept).

Example 7
Find the linear equation in Slope-Intercept Form which contain coordinates $(-5, -3)$ and $(6, 8)$.

$$m=\frac{\Delta y}{\Delta x}=\frac{11}{11}=1$$

$$y-8=1(x-6)$$

$$y=1x-6+8$$

$$y=x+2$$

Example 8
Find the linear equation in Slope-Intercept Form which contain coordinates $(1, 3)$ and $(3, 7)$.

$$m=\frac{\Delta y}{\Delta x}=\frac{4}{2}=2$$

$$y-7=2(x-3)$$

$$y=2x-6+7$$

$$y=2x+1$$

Slope-Intercept Form may be refined more by isolating the y-intercept, keeping x positive, and removing all fractions.

$$y=mx+b \quad \Rightarrow \quad -mx+y=b \quad \Rightarrow \quad -(\text{common denom.})(-mx+y)=b(\text{common denom.})$$

$$\Rightarrow \quad Ax+By=C$$

This type of linear equation is known as Standard Form $Ax+By=C$, where A must be a whole number and B and C must be integers. A or B may be zero but not both. When $A=0$ the line is horizontal and has a slope of zero, when $B=0$ the line is vertical and has a slope of undefined.

Example 9

Find the linear equation in Slope-Intercept Form which contain coordinates $(2, 4)$ and $(4, 5)$.

$$m=\frac{\Delta y}{\Delta x}=\frac{1}{2}=\frac{1}{2}$$

$$y-4=\frac{1}{2}(x-2)$$

$$y=\frac{1}{2}x-1+4$$

$$y=\frac{1}{2}x+3$$

$$-\frac{1}{2}x+y=3$$

$$(-2)\left(-\frac{1}{2}x+y\right)=3(-2)$$

$$x-2y=-6$$

Example 10

Find the linear equation in Slope-Intercept Form which contain coordinates $(1, 3)$ and $(4, 5)$.

$$m=\frac{\Delta y}{\Delta x}=\frac{2}{3}$$

$$y-5=\frac{2}{3}(x-4)$$

$$y=\frac{2}{3}x-\frac{8}{3}+\frac{5}{1}$$

$$y=\frac{2}{3}x+\frac{7}{3}$$

$$-\frac{2}{3}x+y=\frac{7}{3}$$

$$(-3)\left(-\frac{2}{3}x+y\right)=\frac{7}{3}(-3)$$

$$2x-3y=-7$$

Beyond these three major types of linear equations there are two additional special linear equations we must consider. Parallel linear equations must have the same slopes and never touch. Perpendicular linear equations must have opposite reciprocal (opposite sign and flipped fraction) slopes and intersect at a 90 degree angle. Sometimes in Calculus perpendicular lines are referred to as normal lines.

Example 11

Find the linear equation that is parallel to $y=\frac{2}{3}x+4$ that goes through the coordinate $(5, 1)$.

$$y=\frac{2}{3}x+b$$

$$1=\frac{2}{3}(5)+b$$

$$b=-\frac{7}{3}$$

$$y=\frac{2}{3}x-\frac{7}{3}$$

Example 12

Find the linear equation that is perpendicular to $y=\frac{2}{3}x+4$ that goes through the coordinate $(5, 1)$.

$$y=-\frac{3}{2}x+b$$

$$1=-\frac{3}{2}(5)+b$$

$$b=\frac{17}{2}$$

$$y=-\frac{3}{2}x+\frac{17}{2}$$

There are times when you're given data and asked to find a linear regression (linear model) to fit your data. Your calculator has this function built in as LinReg(ax+b) and LinReg(a+bx). Depending on your calculator it will also tell you your correlation coefficient denoted by r. The closer this number is to -1 or 1, the closer the data resembles a linear model. If your calculator does not tell you r, you will need to press 2^{ND} and CATALOG (number 0) and choose DiagnosticOn then press ENTER.

Example 13

The median price of existing single-family homes has increased consistently during the past two decades. However, the data in the table below shows their have been differences in various parts of the country.

Year	Northeast (dollars)	Midwest (dollars)
1970	25,200	20,100
1975	39,300	30,100
1980	60,800	51,900
1985	88,900	58,900
1990	141,200	74,000

a) Find the linear regression equation for the home cost in the Northeast.

We must first type in the data from the table into our calculator. On your TI-83/84 press STAT then choose EDIT. If you have data already in L1, L2, and L3 you will need to clear it by moving your cursor to highlight L1, L2, and L3 and press CLEAR then ENTER. Type the Years into L1, the Northeast into L2, and the Midwest into L3. Now press 2^{ND} and QUIT. Press STAT then cursor to the right for CALC, choose option LinReg(ax+b). You will now see LinReg on your home screen, press 2^{ND} and L1 (number 1) then COMMA (above 7), then 2^{ND} and L2 (number 2), and last press ENTER.

LinReg

$y = ax + b$

$a = 5632$

$b = -11080280$

$r^2 = .9378908639$

$r = .9684476568$

b) What does the slope of the regression line represent?

The slope represents the rate at which the median price is increasing in dollars per year.

c) Find the linear regression equation for the home cost in the Midwest.

From the home screen press STAT then cursor to the right for CALC, choose option LinReg(ax+b). You will now see LinReg on your home screen, press 2^{ND} and L1 (number 1) then COMMA (above 7), then 2^{ND} and L3 (number 3), and last press ENTER.

LinReg

$y = ax + b$

$a = 2732$

$b = -5362360$

$r^2 = .980101269$

$r = .9900006409$

d) Where is the median price increasing more rapidly, in the Northeast or the Midwest?

In the Northeast, the slope (rate of price per year) is greater.

Prerequisites for Calculus - 1.2 Functions and Graphs

Definition: Function

A function comes from a set $D=\{domain\}$ mapped to a set $R=\{range\}$, in which each unique element in the range comes from the domain. Domains may not be repeated, however ranges may be.

$D=\{-1,2,3,5\}$
$R=\{1,4,5\}$ is the same function as $\{(-1, 1), (2, 4), (3, 4), (5, 5)\}$

Since functions are just a set of points, then those points can form an equation. Euler invented the symbolic way $y=f(x)$, this is read "y is a function of x", to identify a function.

Example 1

Identify the domain below. Determine if it is a function.

x	$f(x)$
1	0
2	1
2	3
4	4

Domain $\{1, 2, 4\}$
Not a function.
Domain repeats at 2.

x	$f(x)$
1	5
0	5
2	3
4	3

Domain $\{0, 1, 2, 4\}$
Function.
Domain does not repeat.

Example 2

Identify the domain and range of the functions below. Remember to check for zero on the denominator and under even radicals.

Function	Domain	Range
$f(x)=x$	$(-\infty, \infty)$	$(-\infty, \infty)$
$f(x)=1/x$	$(-\infty, 0)\cup(0, \infty)$	$(-\infty, 0)\cup(0, \infty)$
$f(x)=x^2$	$(-\infty, \infty)$	$[0, \infty)$
$f(x)=x^2+6$	$(-\infty, \infty)$	$[6, \infty)$
$f(x)=\sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$f(x)=\sqrt{x-4}$	$[4, \infty)$	$[0, \infty)$
$f(x)=-\sqrt{x-2}$	$[2, \infty)$	$[0, -\infty)$
$f(x)=\sqrt[3]{x}$	$(-\infty, \infty)$	$(-\infty, \infty)$
$f(x)=\sqrt[3]{x-2}$	$(-\infty, \infty)$	$(-\infty, \infty)$
$f(x)=\sqrt{1-x^2}$	$[-1, 1]$	$[0, 1]$
$f(x)=\dfrac{1}{\sqrt{1-x^2}}$	$(-1, 1)$	$(-1, 1)$

Prerequisites for Calculus - 1.2 Functions and Graphs

> Definition: Even and Odd Functions
> The graphs of even and odd functions have important symmetry properties.
> A function $y = f(x)$ is an
>
> a) even function of x, if $f(-x) = f(x)$
>
> b) odd function of x, if $f(-x) = -f(x)$
>
> for every x in the functions domain.

Example 3

Given the function $f(x) = x^2 + 2$, determine if it is even, odd, or neither.

$f(x) = x^2 + 2 \qquad f(-x) = (-x)^2 + 2$

$f(x) = x^2 + 2 \qquad f(-x) = x^2 + 2$

$f(x) = f(-x)$ for all x, so it is an even function. It also has symmetry about the $y-$axis.

Example 4

Given the function $f(x) = x^3 + x$, determine if it is even, odd, or neither.

$f(x) = x^3 + x \qquad f(-x) = (-x)^3 + (-x)$

$f(x) = x^3 + x \qquad f(-x) = -x^3 - x$

$-f(x) = -x^3 - x \qquad f(-x) = -x^3 - x$

$-f(x) = f(-x)$ for all x, so it is an odd function. It also has symmetry about the origin.

Example 5

Given the function $f(x) = x - 3$, determine if it is even, odd, or neither.

$f(x) = x - 3 \qquad f(-x) = (-x) - 3$

$f(x) = x - 3 \qquad f(-x) = -x - 3$

$-f(x) = -x + 3 \qquad f(-x) = -x - 3$

$-f(x) \neq f(-x)$ for all x, so it is neither an even or odd function.

> Definition: Piecewise Functions
> Two or more formulas each over a different domain combined to form one function. This function can be continuous or discontinuous.

Example 6

$$y = f(x) = \begin{cases} 2, & x < 0 \\ x + 2, & 0 \le x \le 4 \\ -x^2 + 22, & x > 4 \end{cases}$$

Prerequisites for Calculus - 1.2 Functions and Graphs

Example 7

Write the function $y = |x|$ as a piecewise function.

$$y = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

Example 8

Write the function $y = \dfrac{x^2 - 4}{x+2}$, as a continuous piecewise function.

$y = \dfrac{x^2 - 4}{x+2}$, is discontinuous at $x = -2$ because of a "hole". If you reduce the original equation by canceling out the in common factor, you will see how the function should really be graphed.

$y = \dfrac{(x+2)(x-2)}{x+2} \Rightarrow y = x - 2$, now evaluate it at $x = -2$, to find the y value that will make it continuous, $y(-2) = (-2) - 2 \Rightarrow y = -4$. We can now write our piecewise defined continuous function.

$$y = \frac{x^2 - 4}{x+2} \qquad \Rightarrow \qquad y = \begin{cases} \dfrac{x^2 - 4}{x+2}, & x \neq -2 \\ -4, & x = -2 \end{cases}$$

Definition: Composite Functions

A composite function is a function of two or more functions, where the output of one function becomes the input of another function.

$f(g(x))$ and $f \circ g(x)$ are composite functions (read as "f of g of x", x is the input of the function g, which gives the output $g(x)$. We now use the value $g(x)$ as the input of the function f, which will give the output $f(x)$.

Example 9

Given $f(x) = 2x^2 + 1$ and $g(x) = 3x - 1$, find $f(g(1))$ and $g(f(1))$.

$f(g(1))$

Input 1 into $g(x)$ to find the input for $f(x)$

$g(1) = 3(1) - 1$

$g(1) = 2$

Input 2 into $f(x)$ to find the output for $f(g(1))$

$f(2) = 2(2)^2 + 1$

$f(2) = f(g(1)) = 9$

Example 9 (continued)

$g(f(1))$

Input 1 into $f(x)$ to find the input for $g(x)$

$f(1)=2(1)^2+1$

$f(1)=3$

Input 3 into $g(x)$ to find the output for $g(f(1))$

$g(2)=3(3)-1$

$g(2)=g(f(1))=8$

Example 10

Given $f(x)=2x^2+1$ and $g(x)=3x-1$, find $f(g(x))$ and $g(f(x))$.

$f(g(x))=2(g(x))^2+1$

$f(g(x))=2(3x-1)^2+1$

$f(g(x))=2(9x^2-6x+1)+1$

$f(g(x))=18x^2-12x+2+1$

$f(g(x))=18x^2-12x+3$

$g(f(x))=3(f(x))-1$

$g(f(x))=3(2x^2+1)-1$

$g(f(x))=6x^2+3-1$

$g(f(x))=6x^2+2$

Example 11

Given $f(x)=2x+1$, $g(x)=x^2+3x$, and $h(x)=4x$, find $f(h(g(x)))$.

First find $h(g(x))$

$h(g(x))=4(g(x))$

$h(g(x))=4(x^2+3x)$

$h(g(x))=4x^2+12x$

Now, put that into $f(x)$ to find $f(h(g(x)))$.

$f(h(g(x)))=2(h(g(x)))+1$

$f(h(g(x)))=2(4x^2+12x)+1$

$f(h(g(x)))=8x^2+24x+1$

Prerequisites for Calculus - 1.3 Exponential Functions

> Definition: Exponential Function
> Let a be a positive real number other than 1. The function $f(x) = a^x$ is the exponential function with base a.
> If $0 < a < 1$, then it is exponential decay.
> If $a > 1$, then it is exponential growth.

Example 1

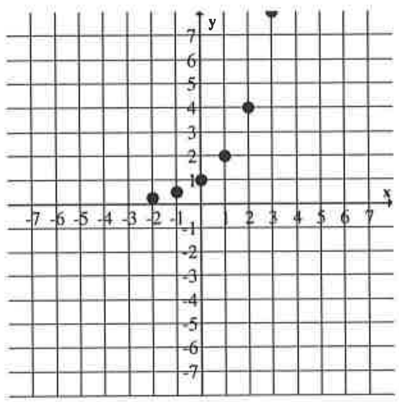
Is $y = 1.1^x$ an exponential growth or decay?

It is an exponential growth because $a > 1$. The y-intercept is $(0, 1)$, when we substitute in $x = 0$.

Example 2

Given the domain $\{-2, -1, 0, 1, 2, 3\}$, find the range for the exponential function $y = 2^x$. Plot the coordinates.

x	$f(x)$
-2	.25
-1	.5
0	1
1	2
2	4
3	8



> Rules for Exponents
> If $a > 0$ and $b > 0$, the following hold for all real numbers x and y.
>
> 1. $a^x \cdot a^y = a^{x+y}$
> 2. $\dfrac{a^x}{a^y} = a^{x-y}$
> 3. $(a^x)^y = (a^y)^x = a^{xy}$
> 4. $a^x \cdot b^x = (ab)^x$
> 5. $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

Example 3

Reduce the following

$2^3 \cdot 4 = 2^3 \cdot 2^2 = 2^{3+2} = 2^5$

$\dfrac{3^3}{3^7} = 3^{3-7} = 3^{-4} = \dfrac{1}{3^4}$

$3(2x^2)^3 = 3 \cdot 2^3 x^{2 \cdot 3} = 24x^6$

$\left(\dfrac{2}{5}\right)^3 = \dfrac{2^3}{5^3} = \dfrac{8}{125}$

Prerequisites for Calculus - 1.3 Exponential Functions

Radioactive Decay
The time it takes for half of a radioactive substance to change it's state to a nonradioactive state by emitting energy in the form of radiation.

Example 4
Suppose the half-life of a certain radioactive substance is 15 days and there are 20 grams present initially. How much is left after 15, 30, and 45 days?
Every 15 days half decays, so after 15 days there is 10 grams, after 30 days there is 5 grams, and after 45 days there is 2.5 grams.

Example 5

If the equation to model the substance above was $y = 20\left(\frac{1}{2}\right)^{\frac{t}{15}}$, how much of the substance is left after 20, 40, and 50 days?

$$y = 20\left(\frac{1}{2}\right)^{\frac{20}{15}} = 7.937 \text{ grams}$$

$$y = 20\left(\frac{1}{2}\right)^{\frac{40}{15}} = 3.150 \text{ grams}$$

$$y = 20\left(\frac{1}{2}\right)^{\frac{50}{15}} = 1.984 \text{ grams}$$

Exponential Rates of Growth

Compounding Interest

$$A(t) = A_0\left(1+\frac{r}{k}\right)^{kt}$$

A_0 is the initial amount, r is the rate, k is the number of times compounded per year, and t is the time in years.

Continuously Compounding Interest

$$P(t) = P_0 e^{rt}$$

P_0 is the initial amount, r is the rate, and t is the time with units that match the r.

Example 6
How long will it take for an investment to double in value if interest is earned at 6.25% compounded monthly and continuously?

$$A(t) = A_0\left(1+\frac{r}{k}\right)^{kt} \quad \Rightarrow 2 = 1\left(1+\frac{0.625}{12}\right)^{12t}$$

$$2 = \left(1+\frac{0.625}{12}\right)^{12t} \quad \Rightarrow \ln 2 = \ln\left(1+\frac{0.625}{12}\right)^{12t}$$

$$\ln 2 = 12t\ln\left(1+\frac{0.625}{12}\right) \Rightarrow \frac{\ln 2}{12\ln\left(1+\frac{0.625}{12}\right)} = t$$

$$t = 11.119 \text{ years}$$

$$P(t) = P_0 e^{rt} \quad \Rightarrow 2 = 1e^{0.0625t}$$

$$2 = e^{0.0625t} \quad \Rightarrow \ln 2 = \ln e^{0.0625t}$$

$$\ln 2 = 0.0625t \ln e \Rightarrow \ln 2 = 0.0625t$$

$$t = \frac{\ln 2}{0.0625} \quad \Rightarrow t = 11.09 \text{ years}$$

Prerequisites for Calculus - 1.5 Functions and Logarithms

Definition: Inverse Functions
If we have a function $f(x)$ and wish to find it's inverse, we must first switch x and y and then solve for y. If f is an invertible function with domain X and range Y, then f^{-1} has domain Y and range X.
Functions are inverses if $f(g(x))=x$ and $g(f(x))=x$ for all x.

Example 1
Find the inverse of $y=x^2+6x+9$
Switch x and y and then solve for y.

$x=y^2+6y+9$

$x=(y+3)^2$

$\pm\sqrt{x}=y+3$

$-3\pm\sqrt{x}=y$

Example 2
Show that $f(x)=2x+4$ and $g(x)=\frac{x}{2}-2$ are inverses.

$f(g(x))=2\left(\frac{x}{2}-2\right)+4$

$f(g(x))=x-4+4$

$f(g(x))=x$

$g(f(x))=\frac{(2x+4)}{2}-2$

$g(f(x))=x+2-2$

$g(f(x))=x$

Example 3
Find the inverse of $y=\frac{x+4}{x-3}$
Switch x and y and then solve for y.

$x=\frac{y+4}{y-3}$

Cross-multiply and distribute, then regroup the y terms.

$x(y-3)=y+4$

$xy-3x=y+4$

$-4-3x=y-xy$

$-4-3x=y(1-x)$

$\frac{-4-3x}{1-x}=y$

Prerequisites for Calculus - 1.5 Functions and Logarithms

Example 4

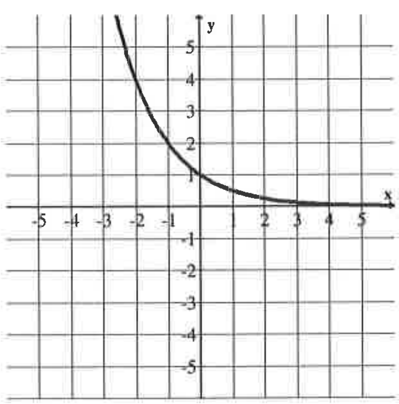
Use parametric equations to graph the inverse of $y = 2^{-x}$

First construct the parametric equations $x(t)$ and $y(t)$, such that when you find the composite $y(x(t)) = y(x)$ the original function.

Let $x(t) = t$ OR Let $x(t) = -t$

Therefore $y(t) = 2^{-t}$ OR Therefore $y(t) = 2^{t}$

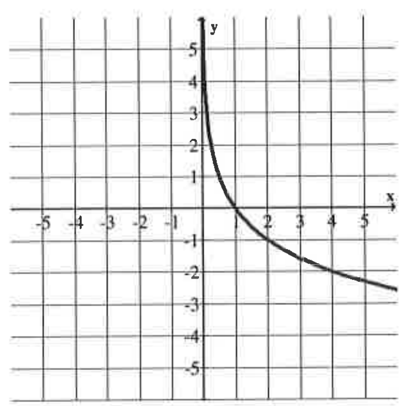
Graph the parametric equation.



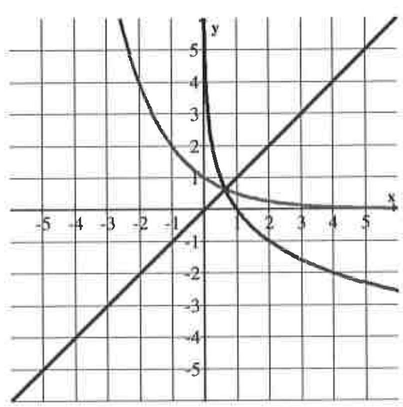
Now to graph the inverse of a parametric equation, simply switch x and y.

$x(t) = t$ becomes $x(t) = 2^{-t}$

$y(t) = 2^{-t}$ becomes $y(t) = t$



Graph of the original function, the inverse function, and the function $y = x$ (mirroring).



Definition: Base a Logarithm Function

The base a logarithm function $y = \log_a x$ is the inverse of the base a exponential function $y = a^x$, where $a > 0$ and $a \neq 1$.

Prerequisites for Calculus - 1.5 Functions and Logarithms

Inverse Properties for a^x and $\log_a x$		
Base a:	$a^{\log_a x} = x$ and $\log_a a^x = x$	for $a > 1$, $x > 0$
Base e:	$e^{\ln x} = x$ and $\ln e^x = x$	for $x > 0$

Properties of Logarithms
For any real numbers $x > 0$ and $y > 0$

Product Rule: $\log_a(xy) = \log_a x + \log_a y$

Quotient Rule: $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

Power Rule: $\log_a x^y = y \log_a x$

Change of Base Formula
$\log_a x = \frac{\log x}{\log a}$ or $\log_a x = \frac{\ln x}{\ln a}$

Example 5
Solve the equation $e^x + e^{-x} = 3$ algebraically.

$e^x + e^{-x} = 3$

$e^x + e^{-x} - 3 = 0$

$e^{-x}\left(e^{2x} + 1 - 3e^x\right) = 0$

$\frac{\left(e^{2x} + 1 - 3e^x\right)}{e^x} = 0$

You can move e^{-x} to the bottom because it never equals zero. Since it never equals zero, it is not a solution and you can remove it by cross-multiplying. You are now left with a "quadratic".

$e^{2x} - 3e^x + 1 = 0$

$\left(e^x\right)^2 - 3e^x + 1 = 0$

We can use variable substitution here to make it look nicer. Let $y = e^x$.

$y^2 - 3y + 1 = 0$

$y = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$

$y = \frac{3 \pm \sqrt{5}}{2}$

Now variable substitute back in.

Replace y with e^x and solve for x.

$y = \frac{3 \pm \sqrt{5}}{2}$

$e^x = \frac{3 \pm \sqrt{5}}{2} \Rightarrow \ln e^x = \ln\left(\frac{3 \pm \sqrt{5}}{2}\right)$

$x = \ln\left(\frac{3 \pm \sqrt{5}}{2}\right)$

Prerequisites for Calculus - 1.5 Functions and Logarithms

Example 6

Sarah invests \$1000 in an account that earns 5.25% interest compounded annually. How long will it take the account to reach \$2500, \$3000, and \$3500? How long did it take from \$2500 to \$3000 and from \$3000 to \$3500?

$$A(t) = A_0\left(1+\frac{r}{k}\right)^{kt}$$
$$2500 = 1000\left(1+\frac{0.0525}{1}\right)^{1t}$$
$$2500 = 1000(1.0525)^t$$
$$2.5 = (1.0525)^t$$
$$\ln 2.5 = \ln(1.0525)^t$$
$$\ln 2.5 = t\ln(1.0525)$$
$$t = \frac{\ln 2.5}{\ln 1.0525} = 17.907 \text{ years}$$

$$A(t) = A_0\left(1+\frac{r}{k}\right)^{kt}$$
$$3000 = 1000\left(1+\frac{0.0525}{1}\right)^{1t}$$
$$3000 = 1000(1.0525)^t$$
$$3 = (1.0525)^t$$
$$\ln 3 = \ln(1.0525)^t$$
$$\ln 3 = t\ln(1.0525)$$
$$t = \frac{\ln 3}{\ln 1.0525} = 21.471 \text{ years}$$

$$A(t) = A_0\left(1+\frac{r}{k}\right)^{kt}$$
$$3500 = 1000\left(1+\frac{0.0525}{1}\right)^{1t}$$
$$3500 = 1000(1.0525)^t$$
$$3.5 = (1.0525)^t$$
$$\ln 3.5 = \ln(1.0525)^t$$
$$\ln 3.5 = t\ln(1.0525)$$
$$t = \frac{\ln 3.5}{\ln 1.0525} = 24.483 \text{ years}$$

How long did it take to get from \$2500 to \$3000?

$$\frac{\ln 3}{\ln 1.0525} - \frac{\ln 2.5}{\ln 1.0525} = \frac{\ln 3 - \ln 2.5}{\ln 1.0525} = \frac{\ln \frac{3}{2.5}}{\ln 1.0525} = \frac{\ln \frac{6}{5}}{\ln 1.0525} = 3.563 \text{ years}$$

or

$$21.471 - 17.907 = 3.564 \text{ years}$$

How long did it take to get from \$3000 to \$3500?

$$\frac{\ln 3.5}{\ln 1.0525} - \frac{\ln 3}{\ln 1.0525} = \frac{\ln 3.5 - \ln 3}{\ln 1.0525} = \frac{\ln \frac{3.5}{3}}{\ln 1.0525} = \frac{\ln \frac{7}{6}}{\ln 1.0525} = 3.013 \text{ years}$$

or

$$24.483 - 21.471 = 3.012 \text{ years}$$

Prerequisites for Calculus - 1.6 Trigonometric Functions

Definition: Periodic Function, Period
A function $f(x)$ is periodic if there is a positive number p such that $f(x+p)=f(x)$ for every value of x. The smallest such value of p is the period.

Example 1
How to finding and graph a trigonometric model.

$y = A\sin\left[\frac{2\pi}{B}(x-C)\right]+D$

$A =$ magnitude

$B =$ period

$C =$ horizontal shift

$D =$ vertical shift

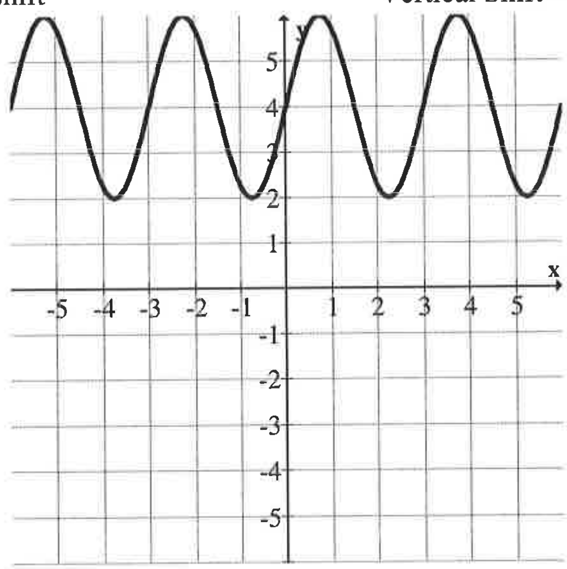
$y = 2\sin\left[\frac{2\pi}{3}(x-4)\right]+1$

Magnitude = up and down 2 from the axis $y = D$

Period = 3

Horizontal Shift = right 4

Vertical Shift = up 1



Definition: Trigonometric Functions

Function	Domain	Range
$y = \cos x$	$(-\infty, \infty)$	$[-1, 1]$
$y = \sin x$	$(-\infty, \infty)$	$[-1, 1]$
$y = \tan x$	$x \neq \frac{\pi}{2} + \pi n$	$(-\infty, \infty)$
$y = \sec x$	$x \neq \frac{\pi}{2} + \pi n$	$(-\infty, -1] \cup [1, \infty)$
$y = \csc x$	$x \neq \pi n$	$(-\infty, -1] \cup [1, \infty)$
$y = \cot x$	$x \neq \pi n$	$(-\infty, \infty)$

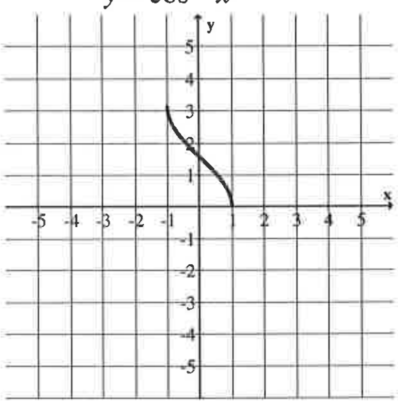
where n is an integer

Prerequisites for Calculus - 1.6 Trigonometric Functions

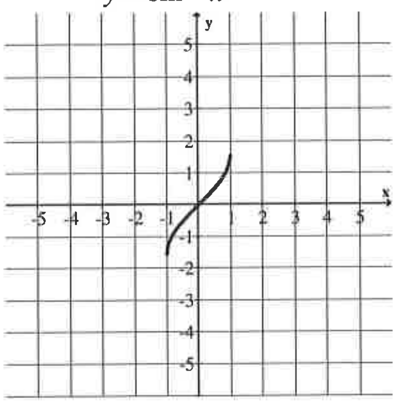
Definition: Inverse Trigonometric Functions

Function	Domain	Range
$y=\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$y=\sin^{-1}x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y=\tan^{-1}x$	$(-\infty, \infty)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y=\sec^{-1}x$	$\lvert x\rvert \geq 1$	$[0, \pi]$, $y \neq \frac{\pi}{2}$
$y=\csc^{-1}x$	$\lvert x\rvert \geq 1$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $y \neq 0$
$y=\cot^{-1}x$	$(-\infty, \infty)$	$(0, \pi)$

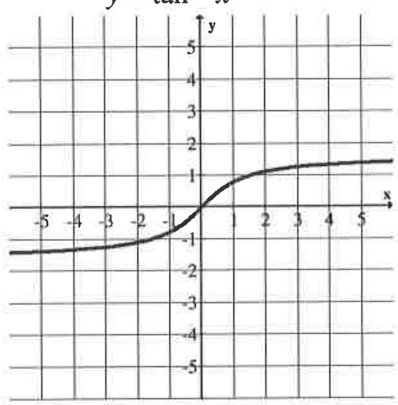
$y=\cos^{-1}x$



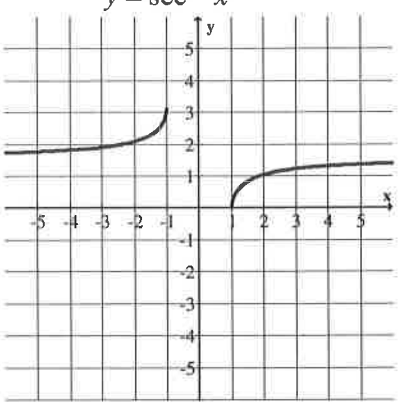
$y=\sin^{-1}x$



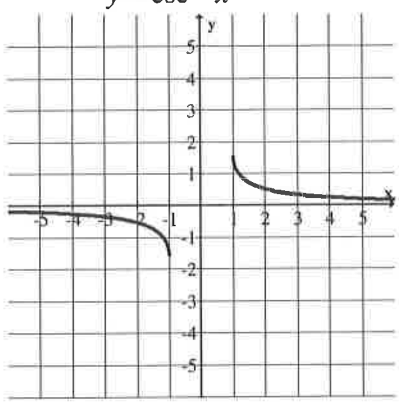
$y=\tan^{-1}x$



$y=\sec^{-1}x$



$y=\csc^{-1}x$



$y=\cot^{-1}x$