Algebra II Honors PLUS
Summer Assignment 2014-2015

Overview

Algebra II Honors PLUS is a course designed to eventually prepare you for AP BC Calculus by creating a strong foundation in algebraic skills and concepts as well as encouraging problem solving and analysis skills. A strong foundation in both Algebra and Geometry is necessary, along with a positive work ethic. I strongly encourage you to come to class prepared by completing all assignments. You will be expected to not only be proficient with your Algebra skills but to also be efficient as well. A thorough understanding of the material means you can not only solve the problem but that you can solve it quickly. Your work and solutions should also demonstrate that you have a strong grasp of the mathematical language and you should be using correct mathematical notations. The following assignment will help you review some essential algebraic skills you will need at the beginning of the year and help you review critical algebra I skills. It also includes some new concepts and material that you should study and be familiar with before class starts in September. **The assignment will be collected the FIRST day of class.** If you have difficulty with this assignment, be proactive by utilizing a mathematics tutor, study group, or tutorials on the internet to help you prepare for the course. This is especially important for incoming freshmen. You need to determine if you have a weakness in any of the topics covered and try to remedy these weaknesses before the start of the school year. If you are struggling with this assignment that may be an indication that the plus class is not the best fit for your mathematical proficiency and we strongly suggest you consider the Algebra II Honors class instead. It is highly suggested that freshmen complete this summer assignment BEFORE freshman orientation in case you would like to ask questions or get some help.

Learn algebra well, and you will do well in all mathematics courses that follow.

Part I: Algebra Review

Complete the following set of problems. If needed please review your algebra skills using old notes, algebra textbooks, a tutor or any algebra review website similar to: http://www.purplemath.com/modules/index.htm or http://www.coolmath.com/algebra/Algebra1/index.html.

Do your work in pencil, with mistakes cleanly erased, not crossed or scratched out.

Write legibly (suitably large and suitably dark); if the grader can't read your answer, it's wrong.

Show your work. This means showing your steps, not just copying the question from the assignment, and then writing an answer. Show everything in between the question and the answer. Use complete English sentences if the meaning of the mathematical sentences is not otherwise clear. This is extremely important as you prepare for the AP testing environment and how the work for free response questions is graded. For your work to be complete, you need to **explain your reasoning** and make your computations clear.

Do not invent your own notation and abbreviations, and then expect the grader to figure out what you meant. For instance, do not use "#" in your sentence if you mean "pounds" or "numbers". Do not use the "equals" sign ("=") to mean "indicates", "is", "leads to", "is related to", or anything else in a sentence; use actual words. **The equals sign should be used only in equations**, and only to mean "is equal to".

Do not do magic. Plus/minus signs, "= 0", radicals, and denominators should not disappear in the middle of your calculations, only to mysteriously reappear at the end. Each step should be complete and demonstrate good mathematical communication.

Remember to **put your final answer at the end** of your work, and mark it clearly by, for example, underlining it or circling it. Label your answer appropriately; if the question asks for measured units, make sure to put appropriate units on the answer. **If the question is a word problem, the answer should be in words.**

BEFORE STARTING THIS ASSIGNMENT, MAKE SURE YOU HAVE THE LATEST VERSION OF ADOBE READER AND REPRINT. Symbols will be missing or only show as squares if you do not have the latest version. Free reader available here… http://get.adobe.com/reader/

In general, complete this assignment as though you're trying to convince someone that you know what you're talking about. Work should be shown where appropriate, even for multiple choice questions.

1. Which of the following is true given $x < -1$?
   a) The additive inverse of x is less than x
   b) The multiplicative inverse of x is less than x.
   c) The multiplicative inverse of x is greater than the additive inverse of x.
   d) The multiplicative inverse of x is greater than x.
   e) None of these relationships can be determined with the given information.

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2. Which of the following is/are true given the line $y = -3x - 2$?
   a) The y-intercept is negative.
   b) The x-intercept is negative.
   c) The y coordinate of the y-intercept is less than the x coordinate of the x-intercept.
   d) *a)* and *b)* are true, but *c)* is false
   e) *a), b),* and *c)* are true.

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3. Which of the following is/are true given $y > 1$?

a) $(y^3)^{100} = y^3 y^{100}$

b) $(y^3)^{100} > y^3 y^{100}$

c) $(y^3)^{100} < y^3 y^{100}$

d) The relationship cannot be determined without more information.

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4. $\sqrt{5^2 + 4^2} =$

a) 20 *b*) 9 *c*) 81

d) $\sqrt{41}$ *e*) undefined

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5. Expand $(3x - 1)^2(x + 2)$.

a) $9x^3 + 18x^2 + x + 2$ *b)* $9x^3 + 18x^2 - x - 2$ *c)* $9x^3 + 2$

d) $9x^3 + 12x^2 - 11x + 2$ *e)* $9x^4 + 25x^2 + 4$

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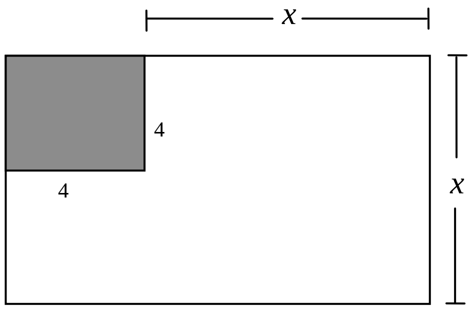
6. A rectangle has a length of $(x - 3)$ and a width of $(3x^2 + 4x)$. What is its perimeter?

a) $3x^3 - 5x^2 - 12x$ *b)* $3x^3 + 4x^2 - 3$ *c)* $3x^2 + 5x - 3$

d) $3x^3 - 12x$ *e)* $6x^2 + 10x - 6$

7. For what value of x will the area of the white region be equal to 61 square inches? Show your algebra equation and all steps used to solve the equation.



Factor each polynomial completely over reals or state prime if the polynomial cannot be factored

8. $6x^2 - 35x - 6 =$ ____________________________

9. $6x^2 + 5x - 6 =$ ____________________________

10. $6x^2 - 13x + 6 =$ ____________________________

11. $9x^2 - 4 =$ ____________________________

12. $9x^2 - 12x + 4 =$ ____________________________

13. $9x^2 + 4 =$ ____________________________

14. $6x^2 - 24x + 24 =$ ____________________________

15. $6x^2 + 24 =$ ____________________________

16. $6x^2 - 24 =$ ____________________________

17. Graph the system of the equations $\begin{cases} x+3y=2 \\ 3x+9y=12 \end{cases}$ and solve using the "substitution" method first the resolve using the "elimination" method. Show all work and steps clearly.

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18. What is the slope of the line perpendicular to the line 5x – 3y + 8 = 0 ? Show work for determining and state reason for selection.

a) $\frac{3}{5}$ b) $\frac{5}{3}$ c) $-\frac{3}{5}$ d) $-\frac{5}{3}$ e) 3

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19. If $R=\frac{ST}{S-T}$, then solve for T and show all steps clearly.

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20. If x = 100, find the value of $\sqrt{\frac{x}{16}-\frac{x}{25}}$. Show all work.

a) 15 b) 5 c) $\frac{5}{2}$ d) $\frac{3}{2}$ e) $\frac{1}{2}$

---

21. What is the slope of the line parallel to the equation $5x + 3y = 2$? Show work for determining and state reason for selection.

a) $-\frac{5}{3}$ b) $-\frac{3}{5}$ c) -5 d) $\frac{5}{3}$ e) 5

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22. What is an equation of the line passing through (3, 0) and (7, 5)?

a) $y=\frac{4}{5}x+3$ b) $y=\frac{4}{5}x+\frac{12}{5}$ c) $y=\frac{2}{3}x$

d) $y=\frac{5}{4}x-3$ e) $y=\frac{5}{4}x-\frac{15}{4}$

Write the equation of this line in point – slope form using the point (7, 5):

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23. What is the equation of the horizontal line that goes through (6, 4)?

a) $x = 4$ b) $x = 6$ c) $y = 4$ d) $y = 6$ e) $y=\frac{3}{2}x$

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24. Given $f(x)=\sqrt{x-2}+\frac{3}{x}$, what is $f(6)$?

a) 2.5 b) 3.5 c) 5.5 d) 6.5 e) 17.5

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25. What is the equation of a line passing through (3, 2) that has undefined slope? What is true about all lines with undefined slope? ____

a) $x = 2$ b) $x = 3$ c) $y = 2$ d) $y = 3$ e) No line exists

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26. What is the equation of a line passing though (3, 2) that has a slope equal to 0? What is true about all lines with a slope of 0? ____

a) $x = 2$ b) $x = 3$ c) $y = 2$ d) $y = 3$ e) No line exists

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For problems 27 – 34, solve for *x*. If more than one solution exists, separate your answers with commas. For example if *x* = 2 or 3, enter 2,3 as your answer. Show all work and intermediate steps clearly.

27. $x^2 + 2x = 15$

$x =$ __________

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28. $3(x-2)^2 = 12$

$x =$ __________

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29. $\dfrac{5}{2x+3} = \dfrac{3}{x}$

$x =$ __________

---

30. $\dfrac{3}{5}x - \dfrac{1}{4}x = 7$

$x =$ __________

---

31. $5[3-(x-2)] = x$

$x =$ __________

---

32. $(x-3)(2x)=0$

$x =$ __________

---

33. $2x^2+4x=0$

$x =$ __________

---

34. $\frac{1}{3}(5x+9)=-2$

$x =$ __________

True or False. Write the letter **T** if the statement is true for all values of *x*. Write the letter **F** if the statement is only true for some values of *x* or not true for any *x*. If FALSE, give a counterexample, a value of x that will make the statement false.

35. $(x)\left(\frac{1}{x}\right)=1,\ where\ x\neq 0.$

36. $x+-x=1$

37. $x>|x|$

38. $\frac{(-3x)^2}{-3x^2}=1$

39. $\frac{x}{1+\frac{1}{3}}=\frac{4}{3}x$

40. $|x|=|-x|$

41. $-x<0$

42. **Solve the system using the "elimination" method showing all steps clearly.**

$$\begin{cases} 4x - 3y = 5 \\ 3x + 2y = 8 \end{cases}$$

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43. If $x = 3$, find the value of $\left(\sqrt{\dfrac{x^2}{16}}\right)\left(\sqrt{\dfrac{4x^2}{25}}\right)$

a) $\dfrac{3}{10}$ b) $\dfrac{9}{10}$ c) $\dfrac{27}{100}$ d) $\dfrac{39}{20}$ e) $-\dfrac{9}{10}$

44. If $x = -3$, find the value of $\left(\sqrt{\dfrac{x^2}{16}}\right)\left(\sqrt{\dfrac{4x^2}{25}}\right)$

a) $\dfrac{3}{10}$ b) $\dfrac{9}{10}$ c) $\dfrac{27}{100}$ d) $\dfrac{39}{20}$ e) $-\dfrac{9}{10}$

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45. $\sqrt{125} + \sqrt{27} - \sqrt{12}$ is equal to which of the following? Show all work.

a) $5\sqrt{5} + \sqrt{3}$ b) $5\sqrt{5} + \sqrt{15}$ c) $5\sqrt{5} - \sqrt{3}$

d) $8\sqrt{8} - 2\sqrt{3}$ e) $6\sqrt{5}$

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46. If the point $\left(-3, \frac{1}{2}\right)$ lies on the graph of the equation $2x + ky = -11$, find the value of k. Show all work and steps below.

a) $-\frac{5}{2}$ b) -34 c) $-\frac{17}{2}$ d) -10 e) -5

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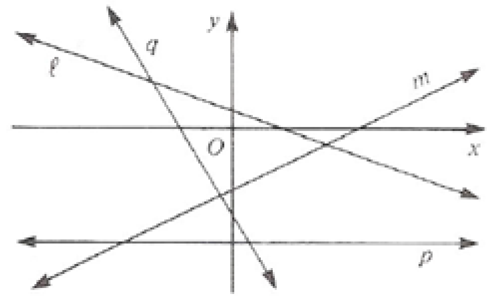
47. Arrange the lines l, m, p, and q in order of increasing slope.



a) *qlpm* b) *lqpm* c) *qlmp*

d) *plmq* e) *pmlq*

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48. Find 3 consecutive integers whose sum is 816. Show algebraic equation used and all steps for solving.

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49. Solve for x: $y = mx + b$

a) $x = \frac{y-b}{m}$ b) $x = \frac{b+y}{m}$ c) $x = \frac{y}{m} + b$ d) $x = \frac{y}{m} - b$

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50. Write an equation that describes the pattern shown below

x	2	3	4	5	6
y	1	-1	-3	-5	-7

a) $y = 2x$ b) $y = 0.5x$ c) $y = -0.5x + 2$
d) $y = -2x + 5$ 3) $y = -2x + 1$

Part 2: Set Properties and Notation

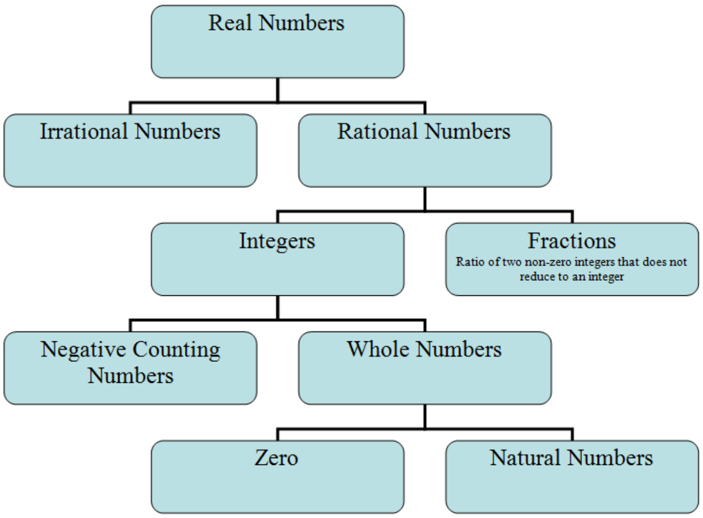
READ THE NOTES BELOW AND COMPLETE ALL PROBLEM SETS.

A. Special Sets

A set is a group or collection of objects that are called members or elements of the set. -2 is an element of the set of real numbers but not a natural number. In mathematical notation we would say this as $-2 \in \mathbb{R}$ and $-2 \notin \mathbb{N}$. If every member of a set A is also a member of a set B, then we say A is a subset of B. or in mathematical notation as $A \subseteq B$. For example, $\mathbb{N} \subseteq \mathbb{R}$. All sets are subsets of themselves, $\mathbb{N} \subseteq \mathbb{N}$. A proper subset cannot be the same set, $\mathbb{N} \subset \mathbb{R}$ but $\mathbb{N} \not\subset \mathbb{N}$ (notice no line underneath to show not equal to). If a set has no elements, it is called the empty set or the null set and it's mathematical notation is $\varnothing$ or $\{\ \}$. There are many sets of numbers that are important in mathematics. Read the following link on the Evolution of Numbers http://www.mathsisfun.com/numbers/evolution-of-numbers.html and please complete the chart below:

Symbol	Name	Description	Examples
$\mathbb{N}$		Counting Numbers	
$\mathbb{W}$			$0,1,2,3,4,5,\ldots$
$\mathbb{Z}$	Integers		
$\mathbb{Q}$	Rational Numbers		$-17,-\frac{19}{7},0,\frac{1}{3},1.37,3.\overline{6},5$
$\mathbb{I}$		Numbers whose decimal representation does not terminate or repeat.	
$\mathbb{R}$	Real Numbers		

The following chart shows the subset relationships of the real number sets:



The number of elements in a **finite set** is known while the number of elements in an **infinite set** is not known. There are 10 elements in the finite set of integers between 0 and 9 inclusive but there is an infinite or unknown number of elements in the set of real numbers between 0 and 9. Before we begin with some math notation, read the following op ed from Steven Strogatz in the New York Times: http://opinionator.blogs.nytimes.com/2010/05/09/the-hilbert-hotel/ . Be prepared to discuss in class on the first day.

B. Set Notation

Using correct notation is an important aspect of learning the mathematical language. Sets can be described using the following set notations:

1. Special set symbols (used for the most commonly used sets),
2. Roster notation (used for small finite sets or large finite sets or infinite sets with a pattern),
3. Set builder notation (can be used for all sets especially those large finite or infinite sets without a pattern) or
4. Interval notation (this notation is only used for sets describing intervals of real numbers)

Example 1: Special Set Symbols (see chart on page 1 for commonly used special symbols)

- Set of Real Numbers = $\mathbb{R}$
- Set of Real Numbers greater than zero = $\mathbb{R}^{>0}$
- Set of Negative Rational Numbers = $\mathbb{Q}^{<0}$

Example 2: Roster Notation – A list or "roster" of the elements in a set.

Note: 1, 2, 3 is just a list of numbers, but $\{1,2,3\}$ is a set containing the elements 1, 2 and 3.

Curly braces $\{\ \}$ are a special notation to indicate mathematically that the objects are being grouped together in a set.

- Set of integers between 1 and 10 inclusive = $\{1,2,3,4,5,6,7,8,9,10\}$
- Set of integers between -5 and 100 inclusive = $\{-5,-4,-3,\ \ldots\ ,98,99,100\}$
- Set of odd integers = $\{\ldots -5,-3,-1,1,3,5,\ldots\}$

Example 3: Set Builder Notation

Note: Curly braces are still used to indicate that we are talking about a set but instead of listing the objects like a roster, we build the set by stating what properties must be satisfied for a number to be an element of the set.

$\left\{x \mid \text{list of properties object must satisfy to be an element}\right\}$

= the set of all objects x such that the following list of properties must be satisfied in order for x to be an element of the set.

- Set of integers between 1 and 10 inclusive = $\{x | 1 \le x \le 10 \text{ and } x \in \mathbb{Z}\}$
- Set of integers between -5 and 100 inclusive = $\{x | -5 \le x \le 100 \text{ and } x \in \mathbb{Z}\}$
- Set of odd integers = $\{x | x \in \mathbb{Z} \text{ and } x = 2p+1 \text{ when } p \in \mathbb{Z}\}$

Example 4: Interval Notation

Note: Only used for intervals of real numbers! Does not use curly braces but uses square brackets and parentheses. The use of brackets and parentheses with interval notation are very important, for they tell us whether or not the endpoints of the interval are included in the set.

-] and [mean that the endpoint is included in the set.
-) and (mean that the endpoint is not included and we always use parentheses for intervals that go to infinity.
- An interval can be closed on one end,] or [, and open on the other,) or (.
    - Set of real numbers between 0 and 9 inclusive = $[0,9]$
    - Set of real numbers between 0 and 9 including 0 only = $[0,9)$
    - Set of positive real numbers = $(0,\infty)$
    - Set of all real numbers = $(-\infty,\infty)$

C. Set Operations

There are two ways to combine two sets together, intersection $\cap$ and union $\cup$. The intersection of two sets returns a set that contains only the elements that are in both of the original sets and the union of two sets returns a set that contains all of the elements that are in either set. Because intersection means both we consider that to be "and" and since union means either we consider that to be "or".

- $\{1,2,3,4,5,6\}\cap\{4,5,6,7,8,9,10\}=\{4,5,6\}$
- $\{1,2,3,4,5,6\}\cup\{4,5,6,7,8,9,10\}=\{1,2,3,4,5,6,7,8,9,10\}$
- $(1,9)\cap[8,15)=[8,9)$
- $(1,9)\cup[8,15)=(1,15)$
- $\mathbb{Q}\cup\mathbb{I}=\mathbb{R}$

There is also a set operation that allows you to negate a set however you must first define what is called a **Universe** or universal set that contains all possible elements that may be grouped together in a subset. The set you want to negate must be a subset of this universe set. So the **negation of a set** is a set that contains all elements in the universe that are <u>not</u> in the original set. For example, if our universe is real numbers, the negation of the set of rational numbers is the set of irrational numbers. A negation is also called the complement of a set. There are two mathematical notations for this complement of a set. The negation or complement of set A is $\sim A$ or $\overline{A}$. You will also see this used as $A-B$ which is called A complement B and returns a set containing all the elements in A that are not in B. For example, $\mathbb{Z}-\mathbb{N}=\mathbb{Z}^{\le 0}$.

D. Venn Diagrams

For a summary of Venn diagrams and how they are related to sets, Review the following lesson from the website: http://www.purplemath.com/modules/venndiag.htm

Problem Set II: Sets

1) For the following examples:
   - Identify whether the set is finite or infinite and whether the set is discrete or continuous.
   - Write the correct mathematical notation for the set using either one of the three set notations (special set symbols, roster notation or set builder notation) and then if set is a continuous interval of real numbers write the correct mathematical notation using interval notation(if not appropriate to use interval notation, state n/a).

   a) Set of all real numbers

   b) Set of all rational numbers

   c) Set of positive integers

   d) Even integers between 0 and 10, including 0 and 10

   e) All real numbers between 0 and 10, including the endpoints

   f) All real numbers between - 3 and 7, excluding the endpoints

   g) All real numbers between ½ and 4, only including the endpoint ½

   h) All integers between ¼ and ½

2) Perform the following operations on sets and write one set to represent the combination.

   a) $\mathbb{Q} \cap \sim \mathbb{Q}$

   b) $\mathbb{Q} \cup \sim \mathbb{Q}$

   c) $\mathbb{Z} - \{n \mid n = 2k \text{ } where\, k \in \mathbb{Z}\}$

   d) $(-\infty, 2) \cap [-5, 20)$

   e) $[-10, 6] \cap (6, 20)$

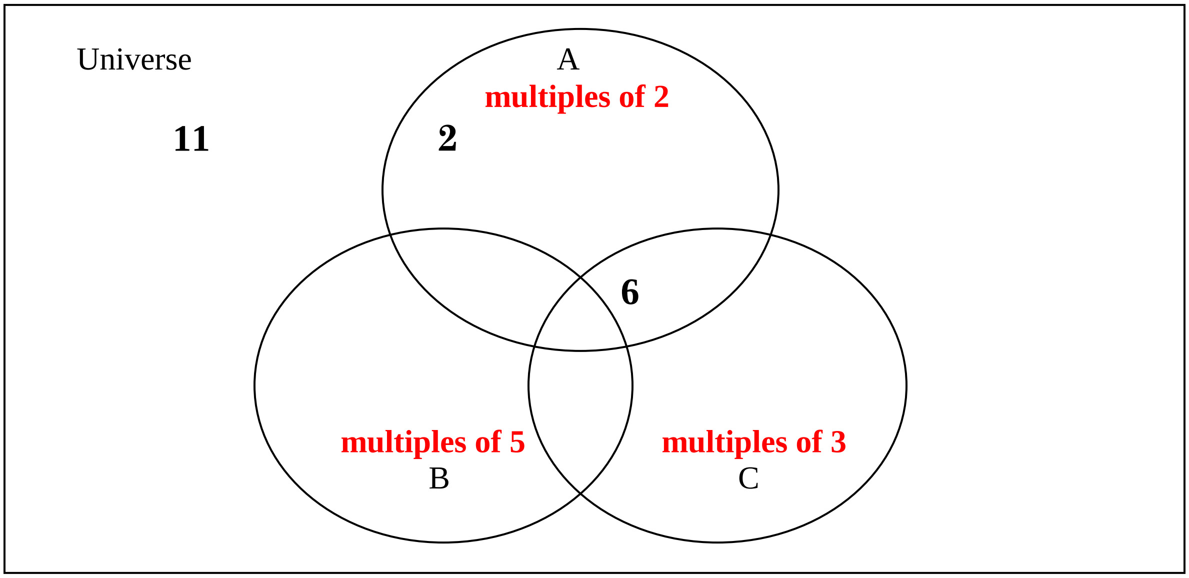
   f) $[-10, 6] \cup (6, 20)$

   g) $(-\infty, 2) - [-5, 20)$

   h) $(-\infty, 2) \cup [-5, 20)$

3) Set Notation and Set Operations

A. Place the integers 2 through 30 inclusive in the appropriate sections of the venn diagram shown below. For example 2, 6 and 11 have already been placed in their correct set.



B. Describe the Universe using set builder notation:

C. Describe the Universe using roster notation:

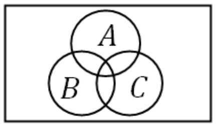
D. Describe set A using set builder notation:

E. Describe sets B using roster notation:

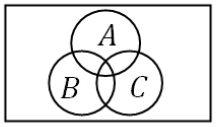
F. Describe the set $A \cup C$ using set builder notation.

G. Describe the set $A \cap C$ using set builder notation.

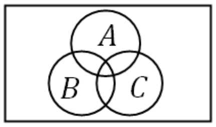
H. For each of the following: Color the appropriate piece on the Venn diagram that corresponds to the given set and describe the set using **<u>roster notation</u>**.



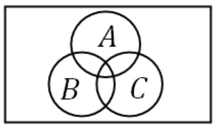
1) $A \cap B =$



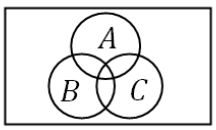
2) $B \cap C$



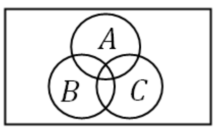
3) $A \cap B \cap C =$



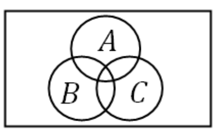
4) $A \cup C =$



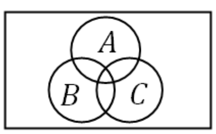
5) $\overline{A} =$



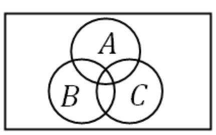
6) $\overline{A} \cap \overline{B} \cap \overline{C}$



7) $(A \cap B) \cup C$



8) $(A - B) \cap C$



9) $(A \cup C) \cap (B \cup C)$

G) Add the following set to your Venn diagram on the previous page.

$S = \left\{ x \middle| x \in Universe \text{ and } x \text{ is a multiple of } 7 \right\}$

H) Add the following set to your Venn diagram on the previous page.

$P = \left\{ x \middle| x \in Universe \text{ and } x \text{ is a prime number} \right\}$

I) Elements and Subsets: Mark each of the following as true or false

____a) $22 \in A$

____b) $15 \in (B \cap C)$

____c) $22 \in S$

____d) $6 \notin P$

____e) $22 \subseteq A$

____f) $\{22\} \subseteq A$

____g) $\{2,3,5\} \not\subseteq P$

____h) $(A \cap B \cap C) \subseteq A$

J) Let the universe be any letter of the alphabet a – m = $\{a,b,c,d,...,k,l,m\}$. Now define new sets A, B and C as follows: $A = \{a,b,c,d\}, B = \{c,d,e,f\}$ *and* $C = \{f,g\}$

Describe each of the following using roster notation:

1) $A \cap B =$

2) $A \cup B =$

3) $A \cap C =$

4) $A \cup B \cup C =$

5) $A - B =$

6) $\overline{A} =$

7) $\overline{B} =$

8) $\{x | x \in A \cup B \, and \, x \, is \, a \, vowel\} =$

9) $\overline{A \cup B} =$

10) $\sim (A \cap B) =$

11) $\overline{A} \cap \overline{B} =$

12) $\overline{A} \cup \overline{B} =$

13) What do you notice about 8 - 11?

14) $A - B =$