Precalculus Plus (Formerly named Trigonometry and Mathematical Analysis Honors PLUS)
Summer Assignment 2018 **Name ______________________________**

Part I: Graphing Parent Functions and the Rule of Four

Quick Review **– The Rule of Four**

There are four basic ways of representing functions: **verbal, algebraic, numerical, and graphical**.

- Verbal includes listing the name of the function and describing its characteristics.
- Algebraic includes writing the equation of the parent function.
- Numerical includes writing 5 key anchor points and features, such as the location of any hole(s) and the equations for all asymptotes (if any). Choose "nice" x values – often we use x values of -2, -1, 0, 1, 2 unless the function is not defined for those values or those values are hard to compute.
- Graphical includes plotting points and sketching the graph. Please indicate your scale on both axes.

Example: Absolute Value Parent Function

Verbal:
Name: *Absolute Value Function*

Domain: $(-\infty,\infty)$ Range: $[0,\infty)$

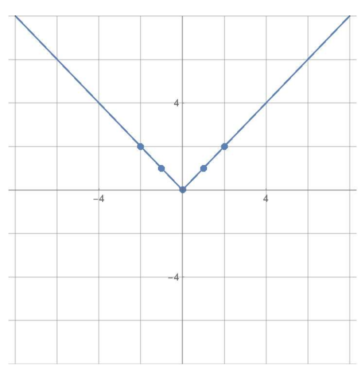
Algebraic (Formula):

$$y=|x|=\begin{cases} x & if \quad x\ge 0 \\ -x & if \quad x<0 \end{cases}$$

Numerical (Anchor Points):

$(-2,2)$
$(-1,1)$
$(0,0)$
$(1,1)$
$(2,2)$

Graphical:



Problem Set I: Graphing Parent Functions and The Rule of Four

Given the name of the following parent functions, complete the rule of four charts for each.

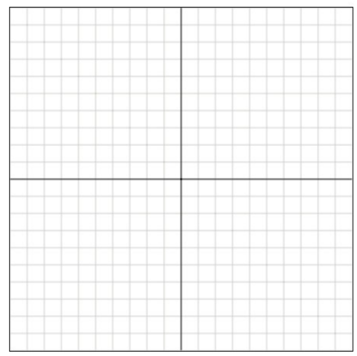
1. Quadratic Function

Verbal:
Name: *Quadratic Function*
Domain: Range:

Algebraic (Formula):

Numerical (Anchor Points):

Graphical:



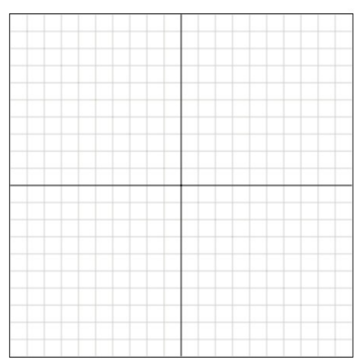
2. Square Root Function

Verbal:
Name: *Square Root Function*
Domain: Range:

Algebraic (Formula):

Numerical (Anchor Points):

Graphical:



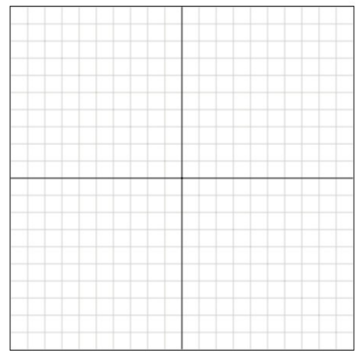
3. Cubic Function

Verbal:
Name: *Cubic Function*
Domain: Range:

Algebraic (Formula):

Numerical (Anchor Points):

Graphical:



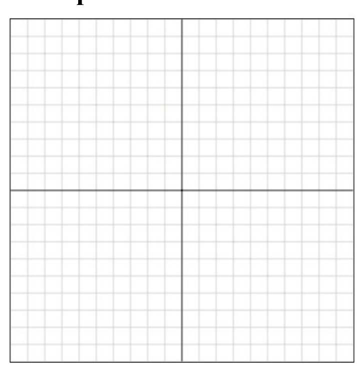
4. Cube Root Function

Verbal:
Name: *Cube Root Function*
Domain: Range:

Algebraic (Formula):

Numerical (Anchor Points):

Graphical:



5. Reciprocal Function

Verbal:

Name: *Reciprocal Function*

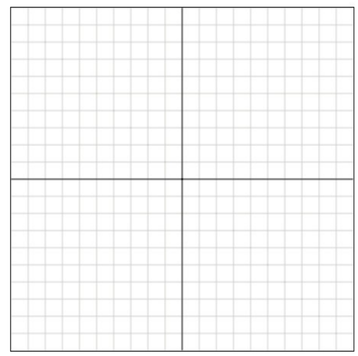
Domain: Range:

Algebraic (Formula):

$$y = \frac{1}{x}$$

Numerical (Anchor Points):

Graphical:



6. Reciprocal Square Function

Verbal:

Name: *Reciprocal Square Function*

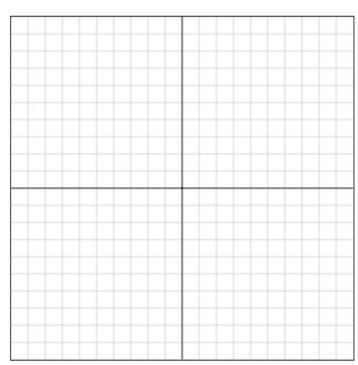
Domain: Range:

Algebraic (Formula):

$$y = \frac{1}{x^2}$$

Numerical (Anchor Points):

Graphical:



Part II: Identifying Characteristics of Functions

Quick Review – Proper Notation

When we refer to a portion of a function, we describe the ***x*-values** for that portion using either Set or Interval Notation:

Set Notation: $\{x | x \geq 3\}$ "The set of all x values such that x is greater than or equal to 3."

Interval Notation: $[3, \infty)$ "The set of all real values of x from 3 to infinity including 3."

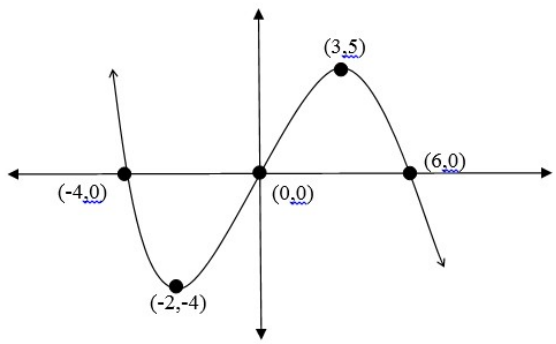
The brackets and parentheses with Interval Notation are very important, for they tell us whether or not the endpoints of the interval are included in the set.

-] and [mean that the endpoint is included and that the function is defined at that point.
-) and (mean that the endpoint is not included or that the function is not defined at that point -- **always use a parenthesis with an infinity symbol**
- An interval can be closed on one end and open on the other.
- $\cup$ is the union symbol and means to include all values from the first interval and all values from the second interval.
- $\cap$ is the intersection symbol and means to include only the values that are in **both** the first interval and the second interval.

Quick Review – Definitions

- A function f is **increasing** on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.
- A function f is **decreasing** on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.
- $f(c)$ is called a **relative (or local) minimum** of the function f, if there is an open interval I containing c on which $f(c) \leq f(x)$ for all x in I.
- $f(c)$ is called a **relative (or local) maximum** of the function f, if there is an open interval I containing c on which $f(c) \geq f(x)$ for all x in I.
- $f(c)$ is called an **absolute (or global) minimum** of the function f if $f(c) \leq f(x)$ for all x in the domain of f.
- $f(c)$ is called an **absolute (or global) maximum** of the function f if $f(c) \geq f(x)$ for all x in the domain of f.
- A smooth, continuous function f is **concave up** at c in its domain if the graph of f lies above the tangent line to f at c.
- A smooth, continuous function f is **concave down** at c in its domain if the graph of f lies below the tangent line to f at c.
- Generally speaking, a function is **continuous** if it does not contain any holes, jumps, or vertical asymptotes.
- Generally speaking, a function is **smooth** if it does not contain any corners or cusps.

Example: Identifying Characteristics of a Function



Characteristics

Increasing: $[-2,3]$

Decreasing: $(-\infty,-2]\cup[3,\infty)$

Constant: Never

Global/Absolute Maximum: None
Global/Absolute Minimum: None

Local/Relative Maximum: point $(3,5)$
A local maximum value of 5 occurs at x = 3.
Local/Relative Minimum: point $(-2,-4)$
A local minimum value of – 4 occurs at x = –2.

Positive: $(-\infty,-4)\cup(0,6)$

Negative: $(-4,0)\cup(6,\infty)$

Zero: $\{x \mid x=-4,0,6\}$

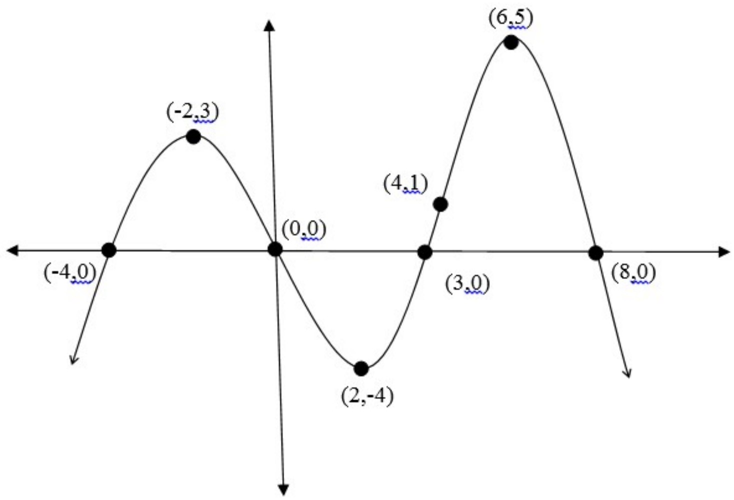
Concave Up: $(-\infty,0)$

Concave Down: $(0,\infty)$

Points of Inflection: $(0,0)$

End Behavior: $f(x)\to-\infty$ as $x\to\infty$
$f(x)\to\infty$ as $x\to-\infty$

Problem Set II: Identifying Characteristics of Functions



Fill in the characteristics for the function above using proper notation. Assume (0, 0) and (4, 1) are the points of inflections.

Characteristics

Increasing:

Decreasing:

Constant:

Global/Absolute Maximum:

Global/Absolute Minimum:

Local/Relative Maximum:

Local/Relative Minimum:

Positive:

Negative:

Zero:

Concave Up:

Concave Down:

Points of Inflection: (0,0) and (4,1)

End Behavior:

<u>Part III: Solving Equations and Inequalities</u>

Students should be able to solve linear, quadratic, polynomial, rational, radical, and absolute value equations and inequalities.

Problem Set III-A: Solve the following equations. You should NOT use a calculator to solve. All solutions should be given as an exact answer (preferably simplified) – NO decimal approximations.

1. $\left[6-4x+2(x-7)\right]-52-3(2x-4)=6\left[3(2x-3)+6\right]$

2. $8-3\left|\frac{1}{2}b-4\right|=2$

3. $-2(2x-3)^2+14=0$

4. $4t^3+4t^2-2t=0$

5. $\frac{x}{x+2}-4=\frac{x+1}{x}$

6. $\dfrac{2x}{x-1}+\dfrac{1}{x-3}=\dfrac{2}{x^2-4x+3}$

7. $2\sqrt{4-y}+10=12$

8. $\sqrt{x+2}=6-\sqrt{7x+2}$

9. $x^{2/3}+3x^{1/3}+2=0$

10. $p(2p-5)^2-3(2p-5)=0$

Problem Set III-B: Solve the following inequalities. Show all work including any sign chart analysis.

11. $-6 \le 1 - 4(x+2) \le 16$

12. $|1-2x| < 4$

13. $|2-5x| > 0$

14. $6x^2 - 7x < 20$

15. $\dfrac{3}{x-2} - \dfrac{1}{x-4} \le 0$