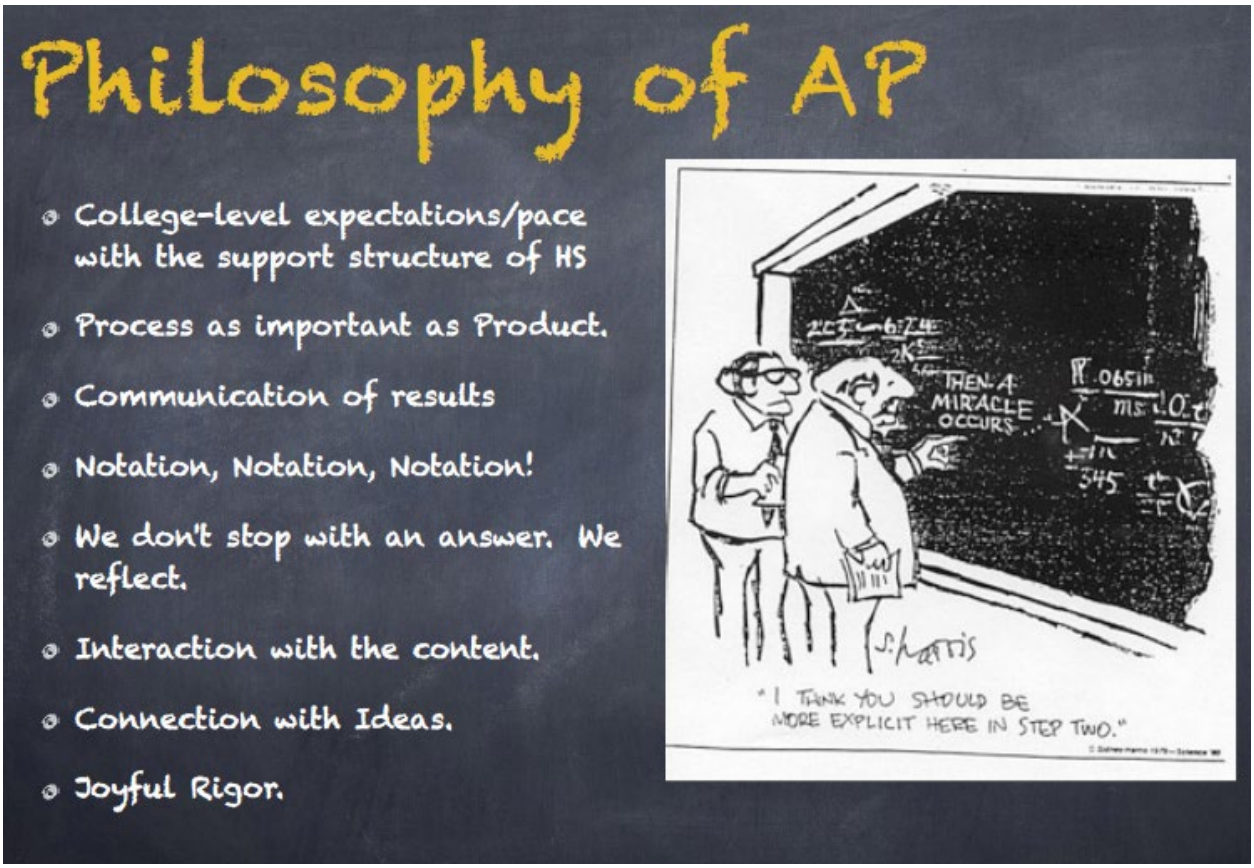
Welcome to AB Calculus

Dear students,

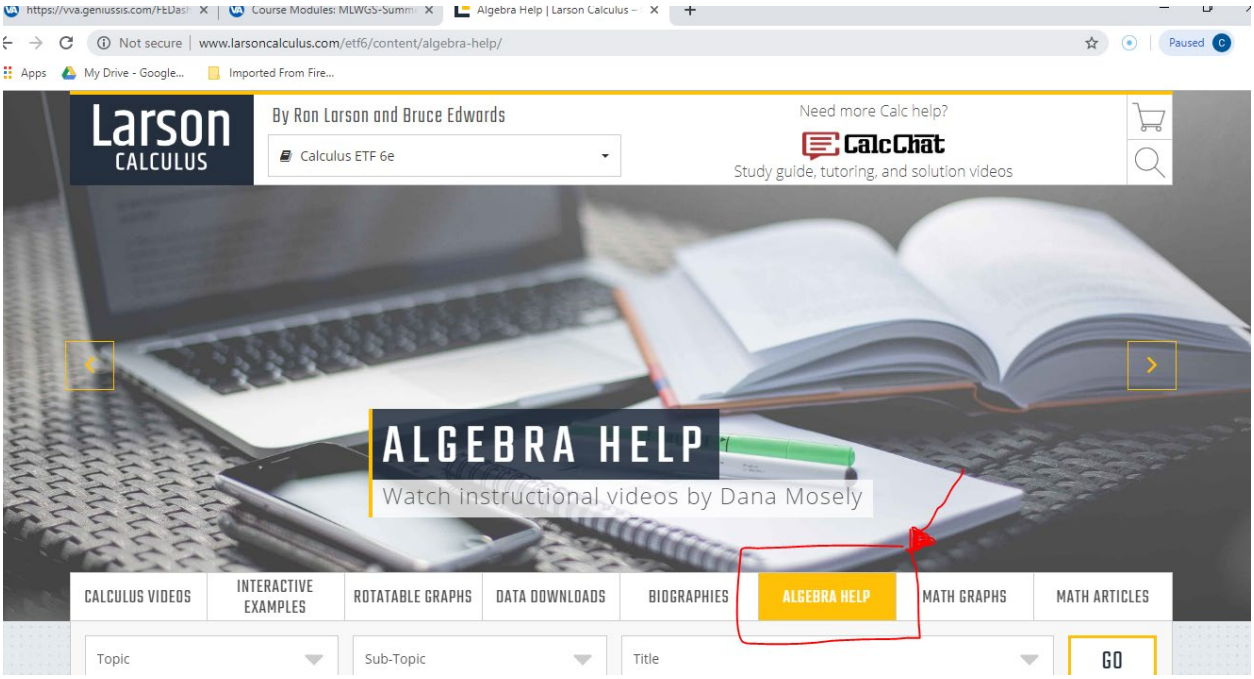
Welcome to AP AB Calculus. I'm excited to be teaching Calculus again this year, seeing a few familiar faces that have been in past classes for Algebra II and Precalculus and meeting new students as well. This will be a fun, interesting and challenging course in which you will learn a lot of mathematics, however mastery of these topics will require hard work and preparation on your part. Be proactive with your learning. Do all of the assigned homework and really focus on understanding and mastery of the concepts. Show your work on all assignments to train yourself for the free response questions on the AP exam. You will be required to not only have an answer, but to also show your work and explain the concept that helps you arrive at your answer. We strive to not only implement mathematical processes but to connect multiple representations of mathematical concepts, communicate using correct notation and mathematical language, and to clearly justify reasonings, use of theorems and explain the meaning of solutions in context. Develop good habits from day 1 in your work throughout the year. Ask lots of questions and come to tutoring if you are struggling with a concept. Don't be afraid to come to me for help as we work together to prepare for the AP exam in May.



It is important that you work through the summer assignment diligently and prepare for the course by making sure you are well versed in PRECALCULUS topics. We have prepared some review materials for you to revisit Algebra and Precalculus topics as needed.

Summer Assignment

Step 1: Visit the website for our textbook: http://www.larsoncalculus.com/etf6/ Click on the tab "Algebra Help".



This is an overview of Precalculus concepts that are the essential skills to be successful in calculus. I have always thought that students who struggle, struggle more with the past algebra skills than with the concepts covered in calculus. As Alfred Lord Tennyson stated, "*the past is prelude, the best is yet to come*". So use the review materials to help put you in the best position for future success in calculus. If you click on "Topics", you should focus your review to the topics listed below:

1. Functions and Graphs
2. Polynomial and Rational
3. Exponential and Logarithmic
4. Trigonometry
5. Analytic Trigonometry

View the list of subtopics for each of the above concepts. Utilize the online resources to revisit any of these topics as needed. You may also use Khan Academy to review topics you feel that you need extra help with. You may find that after attempting the problems in the summer assignment that you are weak in certain areas. Utilize the videos to revisit for extra help as needed.

Step 2: Once you have completed reviewing topics in the study guide materials, read the instructions for "How to complete a circuit". You will complete 5 circuits reviewing various Precalculus Topics. If you get stuck, revisit the review materials. SHOW YOUR WORK. The answer is not the important objective of this review assignment but HOW you worked the problem. Even if it is a calculator problem, state what equation you used the calculator to solve or what expression you used the calculator to find. All problems should have work shown. You may work on your own paper or print.

Step 3: Log into schoology before the first day of class. Upload each circuit to the schoology assignment for that circuit. This must be done by 11:59 pm the day before the first class. Please bring your actual paper work for completed circuits to school on the first day of class for discussions. The first test will cover the material in these topics.

Looking forward to a great year

Crista Hamilton

How to work a circuit

1. Start in the top left box that is already numbered question #1

2. Answer the question
   "Factor the GFC: $24a^2b^3 - 56ab^2$"

   Once you have the answer $8ab^2(3ab-7)$, look through the worksheet for <u>one</u> of the two factors (per the directions)

3. This now becomes problem #2. Fill 2 in on the # _____

4. Repeat the process. Question #2 says, "factor the trinomial $a^2 - 10a + 21$ so that is the product of two binomials".

   $$a^2 - 10a + 21$$
   $$(a-7)(a-3)$$

5. This now becomes problem #3. Fill 3 in on the # _____

6. Repeat the process until your last answer takes you back to the first box. If you have done them all correctly, you will have used them all when you are complete.

Circuit Training – Factoring (Mixed, Intermediate) **Name** _____

Directions: Begin in cell #1. Factor the expression, then search for one of your factors. When you find it, call that problem #2 and continue in this manner until you complete the circuit. You may need to attach additional sheets of paper to showcase your best work.

Answer: _____ # __1__ Factor the GCF: $24a^2b^3 - 56ab^2$	Answer: 2 # _____ If $m = -8$, then there is a unique solution to the equation $x^2 + mx + 16 = 0$. What other value of m yields just one solution?
Answer: $a - 4$ # _____ Factor: $49a^2 + 25b^2$	Answer: $a - 5$ # _____ Factor: $49a^2 - 9b^2$
Answer: $a - 3$ # __3__ Factor by grouping: $ab + 7b + 3a + 21$	Answer: $a^2 - 4a + 16$ # _____ The equation $s(t) = -5t^2 + 3t + 2$ gives the height, $s(t)$, in meters, of a diver at any time t, in seconds, where $t \geq 0$. When does the diver hit the water?
Answer: $4a - 5$ # _____ Use factoring to solve the equation $x^2 - 2x - 3 = 0$. What is the sum of the solutions?	Answer: $a^2 - 5a + 25$ # _____ Factor: $9a^2 - 25b^2$
Answer: $5(a - 1)$ # _____ Simplify: $\frac{a^2-9}{a^2+5a+6}$ for $a > -2$.	Answer: $a + 8$ # _____ Factor: $49a^2 - 14a + 1$
Answer: $3ab - 7$ # __2__ Factor the trinomial $a^2 - 10a + 21$ so that it is the product of two binomials.	Answer: $a - 2$ # _____ Factor: $a^3 - 3a^2 + 5a - 15$

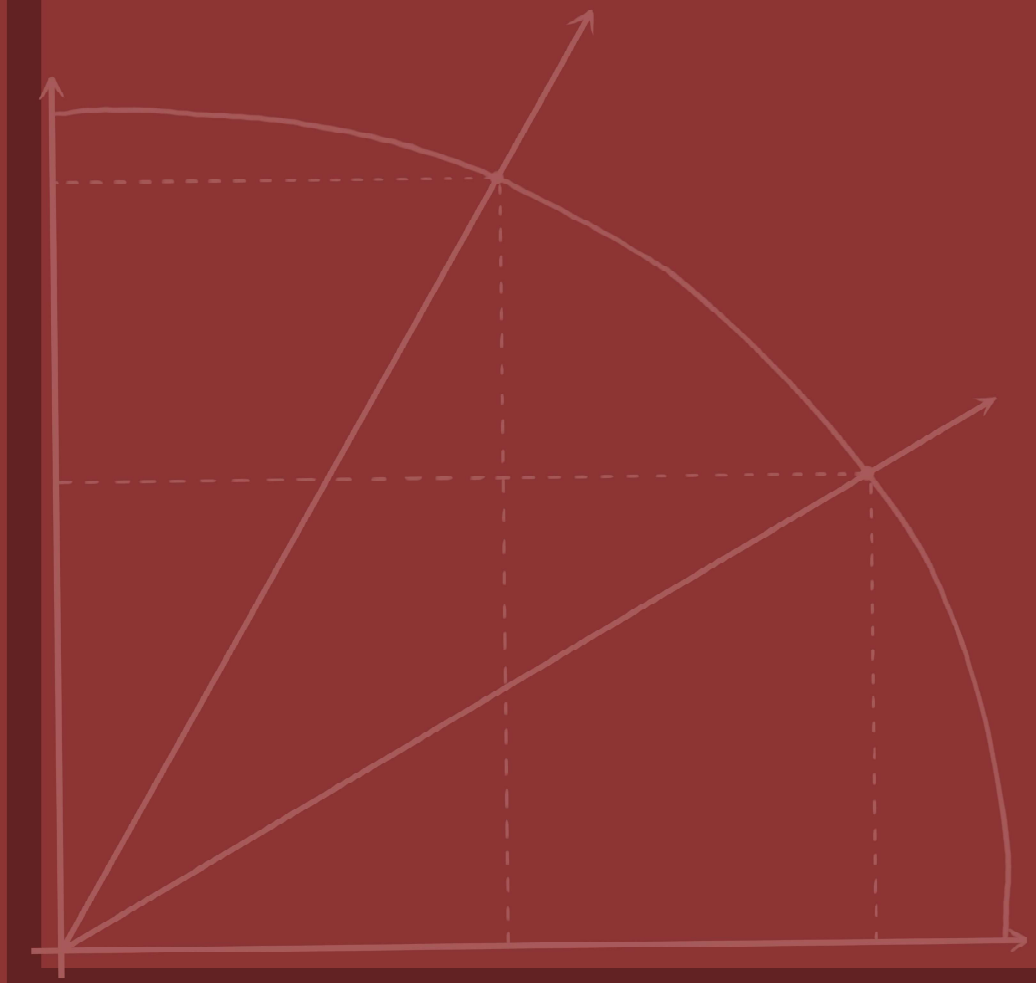
*** For several of the problems you will have to do some simplifying or algebraic manipulation to make it match the answer that is given. ***

Even if you get stuck and can't make the circuit work, you can still do the problems. We will be using circuits regularly.

<u>If you get stuck, pick a new random problem to work and label that #A, work through the circuit from that point. Continue until all problems have been attempted. Highlight the problems that caused issues and use your Precalculus notes and materials to review that concept. We will be discussing on the first day of class.</u>

Generously contributed by Erica Arrington

Precalculus Review



Circuit Training – Factoring (Mixed, Intermediate) **Name** _____

Directions: Begin in cell #1. Factor the expression, then search for *one* of your factors. When you find it, call that problem #2 and continue in this manner until you complete the circuit. You may need to attach additional sheets of paper to showcase your best work.

Answer: _____ # __1__ Factor the GCF: $24a^2b^3 - 56ab^2$	Answer: 2 # _____ If $m = -8$, then there is a unique solution to the equation $x^2 + mx + 16 = 0$. What other value of m yields just one solution?
Answer: $a - 4$ # _____ Factor: $49a^2 + 25b^2$	Answer: $a - 5$ # _____ Factor: $49a^2 - 9b^2$
Answer: $a - 3$ # _____ Factor by grouping: $ab + 7b + 3a + 21$	Answer: $a^2 - 4a + 16$ # _____ The equation $s(t) = -5t^2 + 3t + 2$ gives the height, $s(t)$, in meters, of a diver at any time t, in seconds, where $t \geq 0$. When does the diver hit the water?
Answer: $4a - 5$ # _____ Use factoring to solve the equation $x^2 - 2x - 3 = 0$. What is the sum of the solutions?	Answer: $a^2 - 5a + 25$ # _____ Factor: $9a^2 - 25b^2$
Answer: $5(a - 1)$ # _____ Simplify: $\frac{a^2-9}{a^2+5a+6}$ for $a > -2$.	Answer: $a + 8$ # _____ Factor: $49a^2 - 14a + 1$
Answer: $3ab - 7$ # _____ Factor the trinomial $a^2 - 10a + 21$ so that it is the product of two binomials.	Answer: $a - 2$ # _____ Factor: $a^3 - 3a^2 + 5a - 15$

Answer: $2a-1$ # _____ Rewrite the trinomial $2a^2+13a+15$ as a product of two binomials.	Answer: $7a+2b$ # _____ Factor the difference of squares: a^2-25
Answer: $3a+5b$ # _____ Factor the difference of cubes: a^3b^3-125	Answer: 8 # _____ Factor: a^3+64
Answer: a^2 # _____ Factor: $a^2+16a+64$	Answer: a^2+5 # _____ Factor: $4a^2+7a-15$
Answer: 1 # _____ Factor completely: $2a^3+2a^2-40a$	Answer: $a+5$ # _____ The trinomial x^2-7x-8 can be written as the product of two binomials, $(x+a)(x+b)$. What is $a+b$?
Answer: $\frac{a-3}{a+2}$ # _____ Write a trinomial that has $(3a+17)$ as one of its two factors.	Answer: prime # _____ Use factoring to simplify the rational expression: $\frac{5a^2-5}{a+1}$ (note $a \neq -1$).
Answer: -7 # _____ Factor $21a^4b^2+6a^3b^3$	Answer: $ab-5$ # _____ Factor: $9a^2-25a^2b$
Answer: $7a-1$ # _____ Factor completely: a^4-8a^2+16	Answer: a^3+8 # _____ Factor the sum of cubes: a^3+125
Answer: $7a-3b$ # _10_ Multiply: $(a+2)(a^2-2a+4)$	Answer: $a+7$ # _____ Factor by grouping: $2a^2-14a-1a+7$

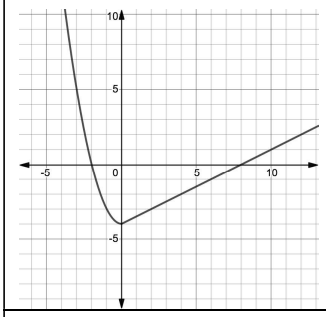
Circuit Training – Piecewise Functions (precalculus) **Name** _____

Directions: Begin in cell #1. Answer the question (show necessary work on this page or attach separate paper). Search for your answer. Call that cell #2 and proceed in this manner until you complete the circuit (get back to the beginning). No technology is needed!

# __1__ Answer: 5 $f(x) = \begin{cases} \lvert 2-x \rvert, & x \le 0 \\ 0.5x^2 - 5, & x > 0 \end{cases}$ $f(6) + f(-3)$	# _____ Answer: 9 $f(n) = \begin{cases} 3n+1, & if\ n\ is\ odd \\ \frac{n}{2}, & if\ n\ is\ even \end{cases}$ $f\left(f\left(f\left(f\left(f\left(f(f(f(42)))\right)\right)\right)\right)\right)$ Look up The Collatz Conjecture if this piques your interest!
# _____ Answer: 8 $f(x) = \begin{cases} -x^3 + 4, & x < -2 \\ -\frac{1}{2}x + 11, & x \ge -2 \end{cases}$ $f^{-1}(10)$	# _____ Answer: 13 Find the minimum value of the function $w(x)$. $w(x) = \begin{cases} x^2 + 4x + 1, & x \le -1 \\ x - 1, & x > -1 \end{cases}$
# _____ Answer: 18 $g(x) = \begin{cases} \frac{x}{2}, & x = -2 \\ 2^{x+3}, & x \ne -2 \end{cases}$ $g(-2) - g(0)$	# _____ Answer: 7 $w(x) = \begin{cases} x^2 + 4x + 1, & x \le -1 \\ x - 1, & x > -1 \end{cases}$ Is $w(x)$ continuous at $x = -1$? If yes, go to answer 10. If no, go to answer 5.
# _____ Answer: -16 Solve $f(x) = 5$. There are four solutions. Find the sum of the solutions. $f(x) = \begin{cases} -x^2 - 6x, & x < 1 \\ \frac{1}{2}x, & x > 1 \\ 5, & x = 1 \end{cases}$	# _____ Answer: 6 $g(x) = \begin{cases} \frac{x+3}{x+2}, & x \ne -2 \\ \frac{1}{4}, & x = -2 \end{cases}$ $g(0) + g(-2) + g(2)$

_____ Answer: -11

The graph shows the piecewise function $f(x) = \begin{cases} x^2 + b\,, x \le 0 \\ ax + c, x > 0 \end{cases}$. Evaluate $\frac{b+c}{a}$.



_____ Answer: -9

$$p(x) = \begin{cases} 9\sin x\,, & x < \frac{\pi}{2} \\ 2 + \cos x\,, & \frac{\pi}{2} \le x < \pi \\ \tan x\,, & x \ge \pi \end{cases}$$

$$\frac{p\left(\frac{2\pi}{3}\right) + p\left(\frac{\pi}{6}\right)}{p\left(\frac{5\pi}{4}\right)}$$

_____ Answer: -3

$$f(x) = \begin{cases} 5e^{x+3}\,, & x \le 0 \\ \ln x\,, & x > 0 \end{cases}$$

$$f(-3) + f(e^3)$$

_____ Answer: 12

$$w(x) = \begin{cases} \frac{|x-5|}{x-5}\,, & x \ne 5 \\ x^3 - 121\,, & x = 5 \end{cases}$$

$$w(2\pi) + w(5) + w(-e)$$

_____ Answer: 10

Find b so that $f(x)$ is a continuous function.

$$f(x) = \begin{cases} bx + 1, & x < 2 \\ \frac{5}{2}x - 6\,, & x \ge 2 \end{cases}$$

_____ Answer: 1

$$p(x) = \begin{cases} 5x - 3, & x < -2 \\ x^2 + 2x + 7, & -2 \le x < 2 \\ x^3 + 8, & x \ge 2 \end{cases}$$

What is the y-intercept of $p(x)$?

_____ Answer: 2

$$f(x) = \begin{cases} x - 5\,, & x < 0.5 \\ 3x + 1\,, & x \ge 0.5 \end{cases}$$

x	-7	-2	2	7
$g(x)$	0.5	-3	0	-6

$$g\big(f(2)\big) + f(g(2))$$

_____ Answer: -1

Find a so that $h(x)$ is a continuous function.

$$h(x) = \begin{cases} \frac{x^2 + 7x - 30}{x - 3}, & x \ne 3 \\ a\,, & x = 3 \end{cases}$$

_____ Answer: 3

$$w(t) = \begin{cases} 4t^2 + 1\,, & t < -1 \\ t + 3\,, & t \ge -1 \end{cases}$$

The equation $w(t) = 17$ has two real solutions. Find the sum of the solutions.

_____ Answer: 4

$$v(t) = \begin{cases} [\![t]\!]\,, & t > 1 \\ |t - 3|\,, & t \le 1 \end{cases}$$

$$v(-2) + v(0) + v(1.42)$$

Circuit Training – Using Tables (pre-calculus) **Name**_____

Directions: The following table shows selected values of three continuous functions f, g, and h. The function h is also strictly decreasing. Beginning in cell #1, use only the values in the table to evaluate the expressions or equations for the given x – value(s). Search for your answer. Call that cell #2 and proceed in this manner until you complete the circuit. For your convenience, the table is on both sides.

Table:

x	$f(x)$	$g(x)$	$h(x)$
0	– 2	3	4
1	3	$\sqrt{2}$	2
2	0	– 3	$\frac{3}{2}$
3	-1	$\frac{\pi}{4}$	0
4	6	$-\frac{4}{3}$	$-\frac{\pi}{2}$
5	7	– 3	– 3

Answer: $-\frac{\pi}{4}$

__1__ $g(5) \cdot h(2)$

Answer: $\frac{3}{2}$

_____ $3g(1) + 2\sin(g(3)) + \cos(h(4))$

Answer: 3

_____ For what value of x does $h(x) = g(x)$?

Answer: $-\frac{9}{2}$

_____ $g(0) - f(1)$

Answer: 4

_____ $\frac{g(3)}{g(4)}$

Answer: $-\frac{3\pi}{16}$

_____ Find $g(h^{-1}(0))$

Answer: $\frac{\pi+16}{4}$

_____ $\frac{f(4)}{h(0)}$

Table:

x	$f(x)$	$g(x)$	$h(x)$
0	– 2	3	4
1	3	$\sqrt{2}$	2
2	0	– 3	$\frac{3}{2}$
3	-1	$\frac{\pi}{4}$	0
4	6	$-\frac{4}{3}$	$-\frac{\pi}{2}$
5	7	– 3	– 3

Answer: $2-\sqrt{2}$

_____ Let $w(x) = e^{h(x)} + 5(f(x))^2$. Find $w(3)$.

Answer: $\frac{\pi}{4}$

_____ If $p(x) = \frac{g(x)+5}{f(x)-6}$, find $p(3)$.

Answer: 0

_____ $f(2) + g(3) + h(0)$

Answer: 5

_____ Let $h^{-1}(x)$ be defined as the inverse of $h(x)$. Find $h^{-1}(2)$.

Answer: 6

_____ Let $r(x) = \sqrt{7 - f(x)}$. Find $r(0)$.

Answer: $4\sqrt{2}$

_____ If $p(x) = h(x) - g(x)$, find $p(1)$.

Answer: $\frac{\pi+20}{-28}$

_____ Find the average rate of change of $h(x)$ on the closed interval $[0, 4]$.

Answer: $-\frac{\pi}{8} - 1$

_____ $\text{Arcsin}\big(f(3)\big) + \text{Arcsec}\,(g(1))$

Answer: 1

_____ For what $x-$value is $p(x) = \frac{g(x)+5}{f(x)-6}$ undefined?

Circuit Training – Precal / Trig Review **NO CALCULATOR!** ☺ Name _____

Directions: Beginning in cell #1, read the question and show the work necessary to answer it (attach separate sheets if necessary). Search for your answer and call that cell #2. Continue in this manner until you complete the circuit. Note: The last question will not have a match!

# 1 Find the slope of the line which connects the point (b, 3b) to the point (3b, 6b). [Note: b≠0.]	Answer: $\frac{-1+ln3}{2}$ # _____ The graph of y = 2 sin(3x - $\frac{\pi}{2}$) has an amplitude of _____, a period of _____, and a phase shift of _____ to the _____ (left/right) when compared to the graph of y = sinx.
Answer: $\frac{2e}{1-e}$ # _____ As x grows infinitely large, the value of $h(x) = \frac{2x}{5x+8}$ approaches what number?	Answer: 4/5 # _____ Find the average rate of change of $w(x) = 3x^2 + 1$ over the interval [-1, 4].
Answer: 75 # _____ For $\frac{\pi}{2} \leq A \leq \pi$, $sinA = \frac{3}{5}$. Find $\sin(2A)$.	Answer: 9 # _____ If $f(x) = lnx$ and $g(x) = e^{x+1}$, find $f(g(2)) - g(f(e))$.
Answer: 21 # _____ $f(x) = g^{-1}(x)$ and $g(x) = \frac{2x}{x-1}$; $f(5) = ?$	Answer: $(-\infty, 2) \cup (2, \infty)$ # _____ $log_{10}25 + log_{10}4 =$
Answer: [-2, 2] # _____ Solve for x: $e^{2x+1} - 3 = 0$	Answer: x = -3 # _____ State the domain of $y = \ln(x - 2)$.
Answer: 2/5 # _____ The expression $3x^2$ is used to calculate the slope at any point on the graph of the function g(x) = $x^3 - 1$. Write the equation of the line tangent to g(x) at its x-intercept.	Answer: 3/2 # _____ The linear function f(x) is parallel to the line $y = \frac{4}{5}x - 7$ and passes through the point (-5, 0). What is f(-6)?

Answer: -4/5 # _____ The quadratic function g(x) has a vertex at (-5, 0) and y-intercept of (0, -5). What is g(1)?	Answer: 2 # _____ The graph of $g(x) = -\sqrt{4-x^2}$ is a semicircle in quadrants III and IV. Find the domain of $g(x)$.
Answer: 4 # _____ Simplify the expression $\frac{x^3+125}{x+5}$ and then evaluate the resulting expression for x = -5.	Answer: 26 # _____ Find $x^2 - y^2$ given that x + y = 7 and x – y = 3.
Answer: $3 - e^2$ # _____ Given $f(x) = x^2 + 5$, find $\frac{f(3+h)-f(3)}{h}$ $(h \neq 0)$.	Answer: 36 # _____ State the range of $w(x) = \frac{2x+1}{x+3}$.
Answer: x > 2 # _____ $81^{\frac{3}{4}} + 8^{\frac{2}{3}} + 125^{\frac{1}{3}}$	Answer: -24/25 # _____ The graphs of $g(x) = \ln(x+3)$ and $f(x) = \frac{2x+1}{x+3}$ have the same vertical asymptote. What is it?
Answer: 5/3 # _____ Solve for x: $\ln(x) - \ln(x+2) = 1$	Answer: y = 3x – 3 # _____ Evaluate g(x) = 5sinx + cos(2x) for x = $\frac{\pi}{2}$.
Answer: -36/5 # _____ Find the average rate of change of the function $p(x) = \frac{4}{5}x - 2$ from x=0 to x=15.	Answer: 6 + h # _____ If the perimeter of a rectangle is 68 and the width is 10, find the length of a diagonal.

Circuit Training – Precalculus /Trig Review **CALCULATOR ACTIVE** Name _____

Beginning in cell #1, use a combination of analytic methods and a graphing calculator to solve the problem. Show how you arrived at your answer, even if a lot of your work was done on the calculator. Hunt for your answer and call this problem #2. Continue in this manner until you complete the circuit. Note: Answers are rounded or truncated to three decimal places. Also, make sure you know HOW to do these on the test when there are no answer choices!

Answer: 0.510 #1 Find the average rate of change for the function $f(x) = 3e^{-x}$ from $x = -1$ to $= 7$.	Answer: 1.771 # _____ The function $r(x) = \frac{x+2}{2x-3}$ has a horizontal asymptote of y = _____.
Answer: -1.750 # _____ Find $f(g\left(-\frac{4\pi}{7}\right))$ if $f(x) = \begin{cases} \lvert x\rvert - 2, & x \le 0 \\ \frac{3}{x}, & x > 0 \end{cases}$ and $g(x) = \tan x$.	Answer: 5.832 # _____ Find the zero of $f(x) = 3 - 2^x$.
Answer: 1.585 # _____ Suppose the number of cases of a rare disease is able to be reduced by 25% annually. If there are 4000 cases nationwide, how many years will it take to reduce the number of cases to 300 ?	Answer: 1.500 # _____ The graph of an exponential function, $y = a \cdot b^x$, passes through the points $(1, 1)$ and $(2, 3.5)$. Find the value of a.
Answer: 0.500 # _____ If $f(g(x)) = g(f(x)) = x$, and $g(x) = 2 + \ln(x + 1)$, find $f(4)$.	Answer: 9.899 # _____ A cone has a height which is one-sixth the radius. If the radius is two, what is the volume of the cone?
Answer: 1.396 # _____ $g(x) = \ln(x - 4)$ and $f(x) = \frac{1}{2}x^2 + 3$. Find $f(g(6))$.	Answer: 0.685 # _____ A drug is administered intravenously for eight hours, $0 \le t \le 8$, and the function $f(t) = 32 - 8.2\ln(1 + 2t)$ gives the number of units of the drug in the body after t hours. How many units are present after 7 hours (at time $t = 7$)?

Answer: 9.004 # _____ What is the period of $y = \sin(4x)$?	Answer: -1.019 # _____ For $g(x) = -3x^2 + 5.2x + 7$, find the maximum value of the function.
Answer: 1.760 # _____ Solve for θ, $\frac{3\pi}{2} \leq \theta \leq 2\pi$. $cos\theta = 0.9$	Answer: 0.456 # _____ What is the minimum value of $y = -3\cos t + 1.25$?
Answer: 9.794 # _____ The function $v(t) = -9.8t + 5$ gives the instantaneous velocity (in m/sec) of an object thrown upward with an initial velocity of 5 m/sec. At what time t does the object start falling?	Answer: 3.240 # _____ Solve the non-linear system $\begin{cases} y = \sqrt{x+2} \\ y = 1.1x^5 \end{cases}$. To advance in the circuit, locate the y-coordinate of the solution.
Answer: 9.253 # _____ An isosceles right triangle has a leg of 7 cm. What is the length of the hypotenuse, in cm?	Answer: 6.389 # _____ Solve $\sec(3x) = 5$ on the open interval $\left(0, \frac{\pi}{6}\right)$.
Answer: 0.286 # _____ $log_3 7$ = ?	Answer: 1.571 # _____ The function $f(x) = \frac{x+2}{2x-3}$ has a vertical asymptote at x = _____.