Algebra II+
Summer Assignment 2023

Dear Student,

Complete the following set of problems. If needed please review your algebra skills using old notes, algebra textbooks, a tutor, or any algebra review website similar to: http://www.purplemath.com/modules/index.htm or https://www.khanacademy.org/math.

Reminders:
Do your work in pencil, with mistakes cleanly erased, not crossed or scratched out.

DO NOT use a calculator.

Write legibly (suitably large and suitably dark); if the grader can't read your answer, it's wrong.

Show your work. This means showing your steps, not just copying the question from the assignment, and then writing an answer. Show everything in between the question and the answer. Use complete English sentences if the meaning of the mathematical sentences is not otherwise clear. This is extremely important as you prepare for the AP testing environment and how the work for free response questions is graded. For your work to be complete, you need to **explain your reasoning** and make your computations clear.

Do not invent your own notation and abbreviations, and then expect the grader to figure out what you meant. For instance, do not use "#" in your sentence if you mean "pounds" or "numbers". Do not use the "equals" sign ("=") to mean "indicates", "is", "leads to", "is related to", or anything else in a sentence; use actual words. **The equals sign should be used only in equations**, and only to mean "is equal to".

Do not do magic. Plus/minus signs, "= 0", radicals, and denominators should not disappear in the middle of your calculations, only to mysteriously reappear at the end. Each step should be complete and demonstrate good mathematical communication.

Remember to **put your final answer at the end** of your work, and mark it clearly by, for example, underlining it or circling it. Label your answer appropriately; if the question asks for measured units, make sure to put appropriate units on the answer. **If the question is a word problem, the answer should be in words.**

1. Makala's solution to an equation is shown below:

   Given: $x^2+5x+6=0$
   Step 1: $(x+2)(x+3)=0$
   Step 2: $x+2=0$ or $x+3=0$
   Step 3: $x=-2$ or $x=-3$

   Which property of real numbers did Makala use for **Step 2**?

   a) Multiplication property of equality
   b) Zero product property of multiplication
   c) Commutative property of multiplication
   d) Distributive property of multiplication over addition

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2. Which equations illustrate the zero property of multiplication? Select **ALL** that apply.

   a) $\frac{1}{3}\cdot 3-3=-2$
   b) $\frac{1}{2}+2-2=\frac{1}{2}$
   c) $0\cdot\frac{1}{9}\cdot 9=0$
   d) $x-5+5=x$
   e) $0\cdot(9+3)=0$

   Name the properties used in the other equations.

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3. Which of the following is/are true given the line $3x+y=6$?
   a) The *y*-intercept is positive.
   b) The *x*-intercept is positive.
   c) The *y* coordinate of the *y*-intercept is less than the *x* coordinate of the *x*-intercept.
   d) a) and b) are true, but c) is false.
   e) a), b), and c) are true.

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4. Evaluate the expression $3^0\left(3^{-2}\right)$

   a) -9
   b) 0
   c) $\frac{1}{9}$
   d) $\frac{1}{81}$

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5. $\sqrt{48}-\sqrt{12}=$

   a) 2
   b) $\sqrt{3}$
   c) $2\sqrt{3}$
   d) $4\sqrt{3}$
   e) 6

6. Expand $(x-3)^2(x+3)$

a) x^3+27x^2+9x+9
b) $x^3-3x^2-18x+9$
c) x^3+27
d) $x^3-3x^2-9x+27$

---

7. Simplify $(4x^4+1)-(7x^3-3)+(2x^4+5x^3)$

a) $6x^4+2x^3-4$
b) $6x^4-2x^3+4$
c) $6x^4-2x^3-2$
d) $2x^4-2x^3-2$
e) $4x+4$

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8. A flower bed is in the shape of a triangle with one side twice the length of the shortest side and the third side 15 feet longer than the shortest side. If the perimeter is 100 feet, and if x represents the length of the shortest side, find an equation to solve for the lengths of the three sides.

a) $x+2x+x+15=100$
b) $x+15=2x$
c) $x+15+2x=100$
d) $x+2x=x+115$

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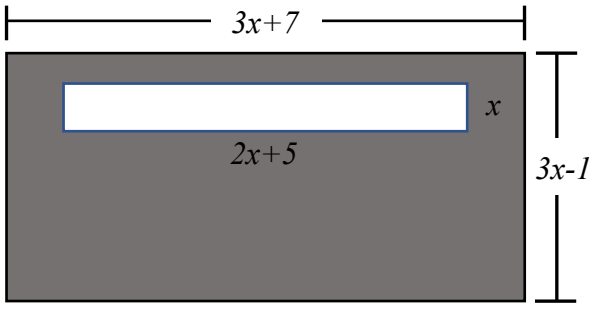
9. In the diagram below, the dimensions of the large rectangle are $(3x-1)$ by $(3x+7)$ units. The dimensions of the cut-out rectangle are x by $2x+5$ units. Which choice expresses the area of the shaded region, in square units?



a) $x^2+23x-7$
b) $x^2+13x-7$
c) $7x^2+23x-7$
d) $7x^2+13x-7$

10. If $B = \frac{2}{5}(A-9)$, then solve for A and show all steps clearly.

11. Identify **ALL** the factors of this polynomial when it is in factored completely.

$$27x^2 - 153x + 90$$

a) 3 b) 9 c) $x-5$ d) $x+5$

e) $3x-2$ f) $3x+2$ g) $3x-15$ h) $9x+6$

12. Four expressions are shown below.

I. $2(2x^2 - 2x - 60)$

II. $4(x^2 - x - 30)$

III. $2(x+6)(x-5)$

IV. $4x(x-1) - 120$

The expression $4x^2 - 4x - 120$ is equivalent to

a) I and II, only
b) II and IV, only
c) I, II, and IV
d) II, III, and IV

13. Which equation shows the correct form of the Quadratic Formula?

a) $x = \frac{b \pm \sqrt{(b)^2 + 4(a)(c)}}{2(a)}$

b) $x = \frac{-b \pm \sqrt{(b)^2 + 4(a)(c)}}{2(a)}$

c) $x = \frac{-b \pm \sqrt{(b)^2 - 4(a)(c)}}{2(a)}$

d) $x = \frac{b \pm \sqrt{(b)^2 - 4(a)(c)}}{2(a)}$

Factor each polynomial completely over reals or state prime if the polynomial cannot be factored

14. $6x^2 - 13x + 5 =$ _____

15. $6x^2 - 7x - 5 =$ _____

16. $6x^2 + 11x + 5 =$ _____

17. $4x^2 - 25 =$ _____

18. $4x^2 - 20x + 25 =$ _____

19. $4x^2 + 25 =$ _____

20. $3x^2 - 12x + 12 =$ _____

21. $3x^2 + 12 =$ _____

22. $3x^2 - 12 =$ _____

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23. Graph the system of equations $\begin{cases} x + 2y = 5 \\ 3x - 2y = 7 \end{cases}$ and solve using the "substitution" method. Show all work clearly.

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24. Which describes the relationship between the two lines?

$y = 2x - 3$

$y = \frac{1}{2}x + 3$

a) The lines are parallel.
b) The lines are perpendicular.
c) The lines intersect but are not perpendicular.
d) The lines are the same.

25. Which pair of lines represent graphs that are perpendicular?

a) $\begin{aligned} y &= -3x+7 \\ y &= -3x+2 \end{aligned}$

b) $\begin{aligned} y &= 5x+5 \\ y &= 10x+5 \end{aligned}$

c) $\begin{aligned} 2y &= 4x-16 \\ y &= 2x-8 \end{aligned}$

d) $\begin{aligned} y &= 9 \\ x &= 5 \end{aligned}$

e) $\begin{aligned} y &= x \\ y &= 3 \end{aligned}$

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26. If $x = 100$, find the value of $\sqrt{\frac{x}{16} - \frac{x}{25}}$. Show all work.

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27. What is the slope of the line parallel to the equation $2x - 3y = 2$? Show work for determining the slope and state your reason for your selection.

a) $\frac{2}{3}$ b) $\frac{3}{2}$ c) -2 d) $-\frac{2}{3}$ e) 2

28.

a) What is an equation of the line passing through $(3,-7)$ and $(24,0)$?

a) $y = 3x + 24$ b) $y = 3x - 72$ c) $y = -\frac{1}{3}x + 8$

d) $y = \frac{1}{3}x - 24$ e) $y = \frac{1}{3}x - 8$

b) Write the equation of this line in point-slope form using the point $(3,-7)$:

29. What is the equation of the vertical line that goes through $(-3,5)$?

a) $x=-3$ b) $x=5$ c) $y=-3$ d) $y=5$ e) $y=-\frac{5}{3}x$

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30. Given $f(x)=\frac{1}{2}x^3+2x$, what is $f(2)$?

a) 5 b) 6 c) 7 d) 8 e) 12

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31. Calculate the slope of the line that passes through the points $(-4,-7)$ and $(1,-7)$.

a) undefined b) 0 c) $\frac{14}{5}$ d) $\frac{14}{3}$ e) No line exists

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32. What is the equation of a line passing through $(6,4)$ that has as slope equal to 0? Complete the following statement: All lines with an undefined slope have _____.

a) $x=6$ b) $x=4$ c) $y=6$ d) $y=4$ e) No line exists

For problems 33 – 41, solve for *x*. List all solutions, if more than one exists. Show all work and intermediate steps clearly.

33. $x^2-8x=-11$

$x=$ _____

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34. $2(x-3)^2=18$

$x=$ _____

35. $\frac{3}{x} = \frac{7}{2x+1}$

$x =$ _____

36. $\frac{1}{2}x - \frac{2}{5} = \frac{1}{10}x + 4$

$x =$ _____

37. $3x + 17 - 5x = 12 - (6x + 3)$

$x =$ _____

38. $x(x+2) = 0$

$x =$ _____

39. $x^3 + 2x^2 - 35x = 0$

$x =$ _____

40. $|3x - 6| = 3$

$x =$ _____

41. $3(x+3) > 4(x-4)$

$x =$ _____

True or False. Write the letter **T** if the statement is true for all values of x. Write the letter **F** if the statement is only true for some values of x or not true for any x. If FALSE, give a counterexample, a value for x that will make the statement false.

42. $4-\frac{b^2}{25}=\left(2-\frac{b}{5}\right)\left(2+\frac{b}{5}\right)$

43. $x^5-3x^2=x^2\left(x^3-3\right)$

44. $\left(x^4\right)^3=x^7$

45. $x^{-4}=\frac{1}{x^4}$

46. $(x)\left(\frac{1}{x}\right)=1,\ \textit{where } x\neq 0$

47. $x+-x=1$

48. $|x|>x$

49. $\frac{(-3x)^2}{-3x^2}=1$

50. $\frac{x}{1+\frac{1}{2}}=\frac{3}{2}x$

51. $|x|=|-x|$

52. $-x<0$

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53. If $y=3x$ and $4x-2y=5$, then $x=$

a) $-\frac{15}{2}$ b) $-\frac{5}{2}$ c) $\frac{11}{6}$ d) $\frac{5}{2}$ e) $\frac{15}{2}$

54. Solve the system of equations using the "elimination" method showing all steps clearly.

$$\begin{cases} 2x-5y=7 \\ 5x+4y=-4 \end{cases}$$

a) $\left(\frac{8}{33},-\frac{43}{33}\right)$

b) $(-5,2)$

c) Infinitely many

d) no real solution

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55. If $x = 3$, find the value of $\left(\sqrt{\frac{x^2}{16}}\right)\left(\sqrt{\frac{4x^2}{25}}\right)$

a) $\frac{3}{10}$ b) $\frac{9}{10}$ c) $\frac{27}{100}$ d) $\frac{39}{20}$ e) $-\frac{9}{10}$

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56. If $x = -3$, find the value of $\left(\sqrt{\frac{x^2}{16}}\right)\left(\sqrt{\frac{4x^2}{25}}\right)$

a) $\frac{3}{10}$ b) $\frac{9}{10}$ c) $\frac{27}{100}$ d) $\frac{39}{20}$ e) $-\frac{9}{10}$

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57. Which polynomial is equivalent to $\frac{8x^3+12x}{2x}$ when $x \neq 0$?

a) $4x^2+6$ b) $4x^2+6x$ c) $4x^2+12x$ d) $4x^4+6x^2$

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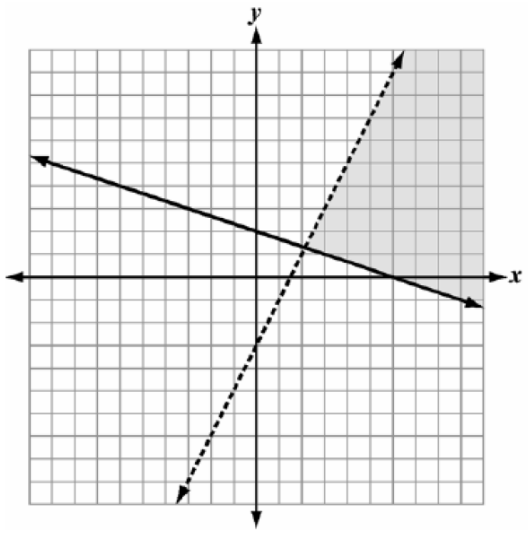
58. Solve for m: $y = mx + b$

a) $m=\frac{y}{x}+b$ b) $m=\frac{b+y}{x}$ c) $m=\frac{y-b}{x}$ d) $m=\frac{y}{x}-b$

59. Which system of inequalities describes the graph?



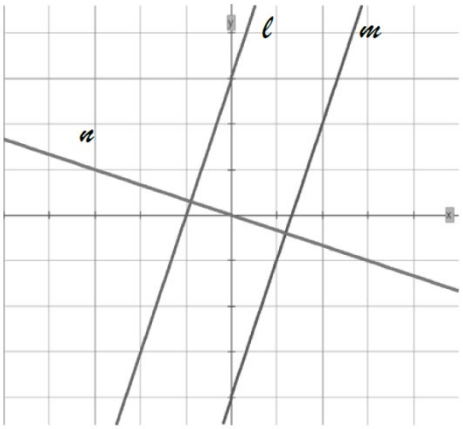
a) $y \geq 2x-3$, $y < -\frac{1}{3}x+2$

b) $y \leq 2x-3$, $y > -\frac{1}{3}x+2$

c) $y > 2x-3$, $y \leq -\frac{1}{3}x+2$

d) $y < 2x-3$, $y \geq -\frac{1}{3}x+2$

For questions 60 – 62, use the graph to the right.



60. The equation for line ℓ is:

a) $y = 3x-4$ b) $y = 3x+3$

c) $y = 3x$ d) $y = 3x-1$

61. The equation for line n is:

a) $y = -\frac{1}{3}x$ b) $y = -\frac{1}{3}x+3$

c) $-\frac{1}{3}x+y=0$ d) $-\frac{1}{3}x-y=2$

62. Which statement is true for lines ℓ, m, and n?

a) ℓ is perpendicular to m

b) ℓ is parallel to n

c) m is parallel to n

d) ℓ is parallel to m

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63. In simplest radical form, $\sqrt[3]{108}$ is equal to

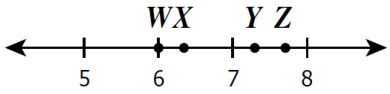
a) $6\sqrt[3]{3}$

b) $3\sqrt[3]{12}$

c) $3\sqrt[3]{4}$

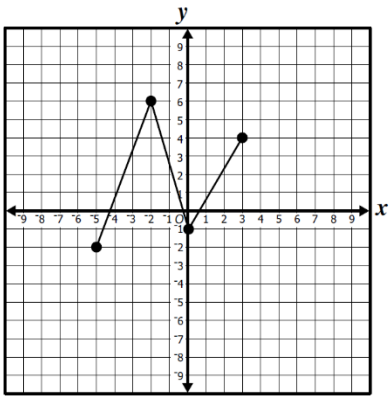
d) 6

64. Which labeled point on the number line is closest to $\sqrt{40}$?



a) W *b) X* *c) Y* *d) Z*

65. What is the domain of the function shown?



a) $-2 \le x \le 6$
b) $-5 \le x \le 3$
c) $-2 \le y \le 6$
d) $-5 \le y \le 3$

66. Which expression is equivalent to $\left(a^3b^4c^6\right)\left(4ac^3\right)^2$?

a) $16a^5b^4c^{12}$ b) $16a^3b^4c^{12}$ c) $8a^3b^4c^{11}$ d) $4a^5b^4c^{12}$

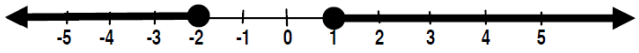
67. Which of the following best describes the circled part of the statement?

$$(7x+9) = 40$$

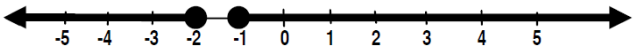
a) Coefficient
b) Variable
c) Term
d) Expression
e) Solution

68. Which of the following graphs represents the solution of $|2x+3| \geq 1$

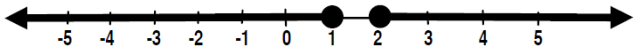
a)



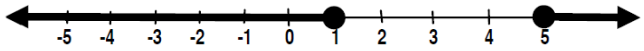
b)



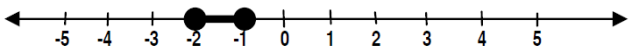
c)



d)



e)



69. The ages of three friends are consecutively one year apart. Together, their ages total 48 years. Which equation can be used to find the age of each friend (where a represents the age of the youngest friend)?

a) $3a = 48$

b) $a(a+1)(a+2) = 48$

c) $a+(a-1)+(a-2) = 48$

d) $a+(a+1)+(a+2) = 48$

What are the ages of the friends?

a) 16,17,18

b) 15,16,17

c) 14,15,16

d) 17,18,19

70. On a scale of 1 to 5 with 1, “I feel very unconfident” and a 5, “I know all of this quickly and readily”, how are you feeling about the material in this assignment?

1 2 3 4 5

Part II: Real Number System and Properties

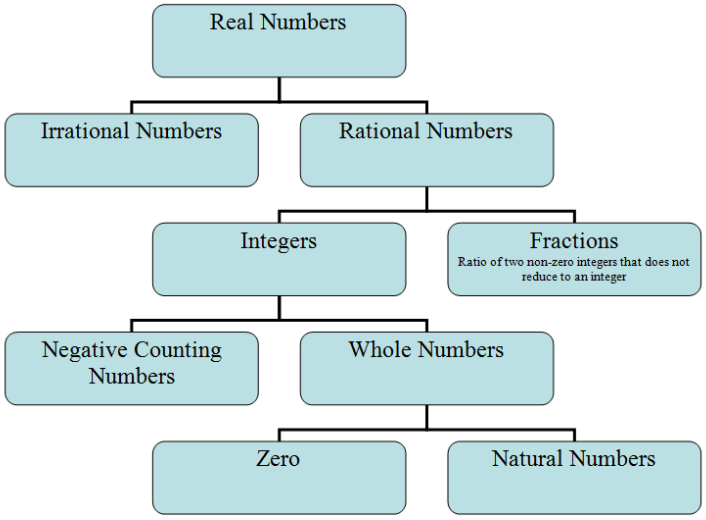
I. Special Sets

A set is a group or collection of objects that are called members or elements of the set. -2 is an element of the set of real numbers but not a natural number. In mathematical notation we would say this as $-2 \in \mathbb{R}$ and $-2 \notin \mathbb{N}$. If every member of a set A is also a member of a set B, then we say A is a subset of B. or in mathematical notation as $A \subseteq B$. For example, $\mathbb{N} \subseteq \mathbb{R}$. All sets are subsets of themselves, $\mathbb{N} \subseteq \mathbb{N}$. A proper subset cannot be the same set, $\mathbb{N} \subset \mathbb{R}$ but $\mathbb{N} \not\subset \mathbb{N}$ (notice no line underneath to show not equal to). If a set has no elements, it is called the empty set, or the null set and its mathematical notation is $\varnothing$ or $\{\ \}$.

There are many sets of numbers that are important in mathematics. Read the following link on the Evolution of Numbers http://www.mathsisfun.com/numbers/evolution-of-numbers.html and please complete the chart below:

Symbol	Name	Description	Examples
$\mathbb{N}$		Counting Numbers	
$\mathbb{W}$			$0,1,2,3,4,5,...$
$\mathbb{Z}$	Integers		
$\mathbb{Q}$	Rational Numbers		$-17, -\frac{19}{7}, 0, \frac{1}{3}, 1.37, 3.\overline{6}, 5$
$\mathbb{I}$		Numbers whose decimal representation does not terminate or repeat.	
$\mathbb{R}$	Real Numbers		

The following chart shows the subset relationships of the real number sets.



Definition: **Rational Number**

If m and n are integers, then $\frac{m}{n}$ is a rational number, provided $n \neq 0$.

If $n = 0$, then $\frac{m}{n}$ would be undefined.

II. **Arithmetic Operations**

Be familiar, able to use, and identify the following properties. There are four binary operations for numbers: **addition, subtraction, multiplication,** and **division**. Addition and multiplication satisfy a set of properties that can be used to change the form of a mathematical **expression** to an equivalent form.

Real Number Properties of Addition and Multiplication

Let $a, b, c \in \mathbb{R}$. Then all of the following are true:

Property		*Addition*	*Multiplication*
Closure	if an operation is applied to 2 or more values from a set of numbers, and the result is also from the same set, it is closed.	$a+b=c$ $a,b,c \in \mathbb{R}$	$ab=c$ $a,b,c \in \mathbb{R}$
Commutative	order of the terms does not matter	$a+b=b+a$	$ab=ba$
Associative	grouping of the terms does not matter	$(a+b)+c=a+(b+c)$	$(ab)c=a(bc)$
Identity	value does not change; 0 is the addition identity element and 1 is the multiplication identity element	$a+0=a$	$a \cdot 1=a$
Inverse	opposite operations	$a+(-a)=0$	$a \cdot \frac{1}{a}=1$
Distributive	Multiplication property over addition and subtraction	$a(b+c)=ab+ac$ $a(b-c)=ab-ac$	

A. Closure Property

Are the following closed given the set of numbers and operation.

1. Given $a, b \in \mathbb{R}$ closed for addition?
2. Given $a, b \in \mathbb{R}$ closed for division?
3. Given $a, b \in \mathbb{N}$ closed for multiplication?
4. Given $a, b \in \mathbb{N}$ closed for subtraction?

III. Properties of Equality

Be familiar, able to use, and identify the following properties.

Equation vs. expression:

Expressions have 1 or more terms. Examples: $x^2+3,\ \ 2x,\ \ 5$

Equations are 2 expressions set equal to each other. Examples: $2x=5,\ \ x^2+3=2x$

** Expressions DO NOT have = signs and are simplified. Equations have = signs and are solved.

Real Number Properties of Equality

Let $a,b,c \in \mathbb{R}$. Then the following properties are true:

Reflexive: $a=a$

Symmetric: $if\ a=b,\ then\ b=a$

Transitive: $if\ a=b,\ and\ b=c,\ then\ a=c$

Addition/Subtraction: $if\ a=b,\ then\ a+c=b+c$ and $a-c=b-c$

Multiplication/Division $if\ a=b,\ then\ ac=bc$ and $\frac{a}{c}=\frac{b}{c}$

Zero Product Property: $if\ ab=0,\ then\ a=0\ or\ b=0$

Zero Property of Multiplication: $a\cdot 0=0$

Substitution: $if\ a+b=c\ then\ a+b+d=c+d$ where $a,b,c,d \in \mathbb{R}$