#### Summer Assignment AP Calculus AB

#### Welcome to AB Calculus

Dear students,

Welcome to AP AB Calculus.

I'm excited to be teaching Calculus again this year.

This will be a fun, interesting and challenging course in which you will learn a lot of mathematics, however mastery of these topics will require



hard work and preparation on your part. Be proactive with your learning. Do all of the assigned homework and really focus on understanding and mastery of the concepts. Show your work on all assignments to train yourself for the free response questions on the AP exam. You will be required to not only have an answer, but to also show your work and explain the concept that helps you arrive at your answer. We strive to not only implement mathematical processes but to connect multiple representations of mathematical concepts, communicate using correct notation and mathematical language, and to clearly justify reasonings, use of theorems and explain the meaning of solutions in context. Develop good habits from day 1 in your work throughout the year. Ask lots of questions and come to tutoring if you are struggling with a concept. Don't be afraid to come to me for help as we work together to prepare for the AP exam in May.

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# Philosophy

- College-level expectations/pace
  with the support structure of HS
- Process as important as Product.
- Communication of results
- Notation, Notation, Notation!
- We don't stop with an answer. We reflect.
- Interaction with the content.
- Connection with Ideas.
- Joyful Rigor.



Students entering AP Calculus need a strong mathematics foundation to have the best opportunity for success in AP Calculus. Below are the skills, content and understandings, along with sample problems, that students should demonstrate from pre-calculus to prepare them for the rigor and challenge of the AP Calculus AB/BC exam.

Multiple Representations—AP Calculus will require students to demonstrate understanding of concepts across multiple representations. Students will see problems in the following representations:

- A. Numerical (through tables)
- B. Graphical (analyzing graphs without equations)
- C. Analytical (algebraic manipulation)
- D. Conceptual (understanding and solving problems at an abstract level)

Additionally, students will need to clearly communicate mathematical thinking and reasoning using precise language with mathematical theorems, definitions and justifications.

These practices should be modeled and utilized in the pre-calculus courses in order to ensure students mastery of material and true understanding of the foundational concepts leading up to AP Calculus. Following are specific content requirements and example questions the model the multiple representation approach above.

#### 1. Write equations of linear functions

Example: Write a linear function y = f(x) in point-slope form with f(2) = 4 and slope of -3.

#### 2. Evaluate functions

Example: Given  $g(x) = x^2 - 3x + 1$ , find g(-1), g(2x-1) and g(x+h)

#### 3. Use composite functions

Example: Given the graphs of f and g below, find f(g(-1))





#### 4. Evaluate piece-wise functions

Example: Given  $h(x) = \begin{cases} x^2 - 4 & x < -1 \\ 3 - \cos(x) & x \ge -1 \end{cases}$ , find h(0).

#### 5. Interpret and Analyze graphs

Examples: The graph of g(x) is below. Solve g(x) = 2.



#### 6. Sketch family of functions (trig, exponentials, polynomials, absolute values, log, rational)

Example: Sketch the parent graphs for  $f(x) = e^x$ ,  $f(x) = \sin x$ ,  $f(x) = \ln(x)$ 

#### 7. Graph transformations

Example: Given y = f(x), describe the graph of g(x) = f(2x) and h(x) = f(x)-1

#### 8. Manipulate formulas

Example: Solve for  $b_1$  given  $A = \frac{1}{2}h(b_1 + b_2)$ 

#### 9. Interpret graphs related to motion (position, velocity, acceleration)

Example: A bug's velocity is given in the graph below. At what time, t, did the bug turn around?



#### 10. Find average Rate of Change

Example: Find the average rate of change of the function  $f(x) = x^3 - x^2 + x - 2$  on the interval [-1, 4].

#### 11. Utilize properties of lines

Example: Solve for "B" given that the lines below are parallel

$$y = 2x - 3, \quad 4x + By = 4$$

#### 12. Perform long division/synthetic division

Example: Use long division to rewrite  $\frac{x^3 - 3x + 1}{2x + 3}$  without an improper fraction.

#### 13. Factor (basic trinomials, greatest common factors, common grouping)

Example: Factor  $-6(x-2)^3(x+3)^4 + 15(x-2)^5(x+3)^2$ 

#### 14. Apply the unit circle

Example: Evaluate  $\tan\left(\frac{5\pi}{6}\right)$ 

#### 15. Find inverse functions

Example: Given 
$$f(x) = \frac{2x-3}{3x+1}$$
, find  $f^{-1}(x)$ 

#### 16. Solve equations without a calculator

Example: Solve for x:  $2x^2 - 5x = x^2 - 3x + 7$ 

#### 17. Use a graphing calculator to solve equations

Example: Use a graphing calculator to solve the equation  $sin(x^2) - e^x = 3.5$ .

18. Use a graphing calculator to graph a function, find an appropriate window, find relative extrema, find points of intersection between two functions, find zeroes, evaluate a function at a given value.

Example: Find the minimum value for  $f(x) = x^4 + 3x^3 - 1.3x - 7.8$ 

These specific skills, knowledge and understandings along with a rigorous pre-calculus course with opportunities for student discourse, higher level thinking and conceptual learning will best prepare students for the challenge and opportunity of an AP Calculus course.

It is important that you work through the summer assignment diligently and prepare for the course by making sure you are well versed in PRECALCULUS topics. We have prepared some review materials for you to revisit Algebra and Precalculus topics as needed.

#### Summer Assignment

#### Step 1: REVIEW AS NEEDED

Visit the website for our textbook: <u>http://www.larsoncalculus.com/etf6/</u> Click on the tab "Algebra Help".



This is an overview of Precalculus concepts that are the essential skills to be successful in calculus. I have always thought that students who struggle, struggle more with the past algebra skills than with the concepts covered in calculus. As Alfred Lord Tennyson stated, *"the past is prelude, the best is yet to come"*. So use the review materials to help put you in the best position for future success in calculus. If you click on "Topics", you should focus your review to the topics listed below:

- 1. Functions and Graphs
- 2. Polynomial and Rational
- 3. Exponential and Logarithmic
- 4. Trigonometry
- 5. Analytic Trigonometry

View the list of subtopics for each of the above concepts. Utilize the online resources to revisit any of these topics as needed. You may also use Khan Academy to review topics you feel that you need extra help with. You may find that after attempting the problems in the summer assignment that you are weak in certain areas. Utilize the videos to revisit for extra help as needed.

#### Step 2: COMPLETE REVIEW CIRCUITS

Once you have completed reviewing topics in the study guide materials, read the instructions for "How to complete a circuit". You will complete 5 circuits reviewing various Precalculus Topics. If you get stuck,

revisit the review materials. **SHOW YOUR WORK USING A PENCIL**. The answer is not the important objective of this review assignment but HOW you worked the problem. Even if it is a calculator problem, state what equation you used the calculator to solve or what expression you used the calculator to find. All problems should have work shown. See end of this document for the attached circuits listed below. You may work on your own paper or print. You must show clear mathematical work to receive full credit.

How to Work a Circuit1
Precalculus Review
Intermediate Factoring
Piecewise Functions
Using Tables
Precal/Trig Review – No Calculator
Precal/Trig Review – Calculator

#### Step 3: BRING TO CLASS ON DAY 1

Bring completed summer assignment to class on day 1. We will discuss in class on Day 1 and start processing.

#### **Step 4:** GRADE, CORRECT AND SUBMIT PDF TO SCHOOLOGY.

As we process your summer work, make corrections using colored markers, pens, or pencils. You will turn in your completed work after grading and correcting with colored marker into Schoology by scanning each circuit individually into a pdf and uploading it to Schoology. We will discuss how to upload during the first class. Upload each circuit to the schoology assignment for that circuit after corrections completed. This must be done by due dates listed in Schoology over the first two weeks of class. The first assessment will cover the material in these topics.

Looking forward to a great year

Crísta Hamílton

How to work a circuit

- 1. Start in the top left box that is already numbered question #1
- 2. Answer the question "Factor the GFC:  $24a^2b^3 - 56ab^2$ "

Once you have the answer  $8ab^2(3ab-7)$ , look through the worksheet for <u>one</u> of the two factors (per the directions)

- 3. This now becomes problem #2. Fill 2 in on the # \_\_\_\_\_
- 4. Repeat the process. Question #2 says, "factor the trinomial  $a^2 - 10a + 21$  so that is the product of two binomials".

 $a^{2}-10a+21$ (a-7)(a-3)

- 5. This now becomes problem #3. Fill 3 in on the # \_\_\_\_\_
- 6. Repeat the process until your last answer takes you back to the first box. If you have done them all correctly, you will have used them all when you are complete.

Circuit Training - Factoring (Mixed, Intermediate)

Name \_\_\_\_\_

Directions: Begin in cell #1. Factor the expression, then search for one of your factors. When you find it, call that problem #2 and continue in this manner until you complete the circuit. You may need to attach additional sheets of paper to showcase your best work.

Answer	Answer: 2
#1 Factor the GCF: $24a^2b^3 - 56ab^2$	# If $m = -8$ , then there is a unique solution to the equation $x^2 + mx + 16 = 0.$
	What other value of m yields just one solution?
Answer: $a - 4$ # Factor: $49a^2 + 25b^2$	Answer: $a - 5$ # Factor: $49a^2 - 9b^2$
Answer: a - 3 # 3 Factor by grouping: ab + 7b + 3a + 21	Answer: $a^2 - 4a + 16$
	# The equation s(t) = -5t <sup>2</sup> + 3t + 2 gives the height, s(t), in meters, of a diver at any time t, in seconds, where t ≥ 0. When does the diver hit the water?
Answer: $4\alpha - 5$ # Use factoring to solve the equation $x^2 - 2x - 3 = 0.$	Answer: $a^2 - 5a + 25$ # Factor: $9a^2 - 25b^2$
What is the sum of the solutions?	
Answer: $5(a-1)$ # Simplify: $\frac{a^2-9}{a^2+5a+6}$ for $a > -2$ .	Answer: α + 8 # Factor: 49α <sup>2</sup> - 14α + 1
Answer: 3ab – 7	Answer: a – 2
# Factor the trinomial a <sup>2</sup> - 10a + 21 so that it is the product of two binomials.	# Factor: a <sup>3</sup> - 3a <sup>2</sup> + 5a - 15

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\*\*\* For several of the problems you will have to do some simplifying or algebraic manipulation to make it match the answer that is given. \*\*\*

Even if you get stuck and can't make the circuit work, you can still do the problems. We will be using circuits regularly.



Section Art by HH Fisher

## **Circuit Training – Factoring (Mixed, Intermediate)**

Name \_\_\_\_\_

<u>Directions</u>: Begin in cell #1. Factor the expression, then search for <u>one</u> of your factors. When you find it, call that problem #2 and continue in this manner until you complete the circuit. You may need to attach additional sheets of paper to showcase your best work.

A	Answer: 2
Answer:	# If $m = -8$ , then there is a unique solution to the equation
#1 Factor the GCF: $24a^2b^3 - 56ab^2$	$x^2 + mx + 16 = 0.$
	What other value of <i>m</i> yields just one solution?
Answer: $a-4$	Answer: $a-5$
# Factor: $49a^2 + 25b^2$	# Factor: $49a^2 - 9b^2$
Answer: $a - 3$	Answer: $a^2 - 4a + 16$
# Factor by grouping: $ab + 7b + 3a + 21$	
	# The equation $s(t) = -5t^2 + 3t + 2$ gives the height, $s(t)$ , in meters, of a diver at
	any time $t$ , in seconds, where $t \ge 0$ . When does the diver hit the water?
Answer: $4a - 5$ # Use factoring to solve the equation	Answer: $a^2 - 5a + 25$
$x^2 - 2x - 3 = 0.$	# Factor: $9a^2 - 25b^2$
What is the sum of the solutions?	
Answer: $5(a-1)$	Answer: $a + 8$
# Simplify: $\frac{a^2-9}{a^2+5a+6}$ for $a > -2$ .	# Factor: $49a^2 - 14a + 1$
Answer: $3ab - 7$	Answer: $a - 2$
# Factor the trinomial $a^2 - 10a + 21$ so that it is the product of two binomials.	# Factor: $a^3 - 3a^2 + 5a - 15$

Answer: $2a - 1$	Answer: $7a + 2b$
# Rewrite the trinomial $2a^2 + 13a + 15$ as a product of two binomials.	# Factor the difference of squares: $a^2 - 25$
Answer: $3a + 5b$	Answer: 8
# Factor the difference of cubes: $a^3b^3 - 125$	# Factor: $a^3 + 64$
Answer: $a^2$	Answer: $a^2 + 5$
# Factor: $a^2 + 16a + 64$	# Factor: $4a^2 + 7a - 15$
Answer: 1 # Factor completely: $2a^3 + 2a^2 - 40a$	Answer: $a + 5$ # The trinomial $x^2 - 7x - 8$ can be written as the product of two binomials, (x + a)(x + b). What is $a + b$ ?
Answer: $\frac{a-3}{a-3}$	Answer: prime
# Write a trinomial that has $(3a + 17)$ as one of its two factors.	# Use factoring to simplify the rational expression: $\frac{5a^2-5}{a+1}$ (note $a \neq -1$ ).
Answer: –7	Answer: $ab - 5$
# Factor $21a^4b^2 + 6a^3b^3$	# Factor: $9a^2 - 25a^2b$
Answer: $7a - 1$	Answer: $a^3 + 8$
# Factor completely: $a^4 - 8a^2 + 16$	# Factor the sum of cubes: $a^3 + 125$
Answer: $7a - 3b$	Answer: $a + 7$
# _10 Multiply: $(a + 2)(a^2 - 2a + 4)$	# Factor by grouping: $2a^2 - 14a - 1a + 7$

## **Circuit Training – Piecewise Functions (precalculus)**

Name \_\_\_\_\_

<u>Directions</u>: Begin in cell #1. Answer the question (show necessary work on this page or attach separate paper). Search for your answer. Call that cell #2 and proceed in this manner until you complete the circuit (get back to the beginning). No technology is needed!

#1	Answer: 5	# Answer: 9
f(x) =	$\begin{cases}  2 - x , & x \le 0\\ 0.5x^2 - 5, & x > 0 \end{cases}$ $f(6) + f(-3)$	$f(n) = \begin{cases} 3n+1, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$ $f\left(f\left(f\left(f\left(f\left(f\left(f\left(f\left(f\left(42\right)\right)\right)\right)\right)\right)\right)\right)$
#	Answer: 8	Look up The Collatz Conjecture if this piques your interest!      #    Answer: 13
f(x) =	$\begin{cases} -x^3 + 4, & x < -2 \\ -\frac{1}{2}x + 11, & x \ge -2 \end{cases}$	Find the minimum value of the function $w(x)$ . $w(x) = \begin{cases} x^2 + 4x + 1, & x \le -1 \\ x - 1, & x > -1 \end{cases}$
	$f^{-1}(10)$	
#	Answer: 18	# Answer: 7
g(x)	$= \begin{cases} \frac{x}{2}, & x = -2\\ 2^{x+3}, & x \neq -2 \end{cases}$	$w(x) = \begin{cases} x^2 + 4x + 1, & x \le -1 \\ x - 1, & x > -1 \end{cases}$
	g(-2) - g(0)	Is $w(x)$ continuous at $x = -1$ ? If yes, go to answer 10. If no, go to answer 5.
#	Answer: -16	# Answer: 6
Solve $f(x) = 5$ . T sum of the solution f(x) =	There are four solutions. Find the ons. $= \begin{cases} -x^2 - 6x, & x < 1\\ \frac{1}{2}x, & x > 1\\ 5, & x = 1 \end{cases}$	$g(x) = \begin{cases} \frac{x+3}{x+2}, & x \neq -2\\ \frac{1}{4}, & x = -2 \end{cases}$ $g(0) + g(-2) + g(2)$



## **Circuit Training – Using Tables (pre-calculus)**

Name\_\_\_\_\_

Directions: The following table shows selected values of three continuous functions f, g, and h. The function h is also strictly decreasing. Beginning in cell #1, use only the values in the table to evaluate the expressions or equations for the given x – value(s). Search for your answer. Call that cell #2 and proceed in this manner until you complete the circuit. For your convenience, the table is on both sides.



Table:		Answer: $2 - \sqrt{2}$
x	$f(x) \qquad g(x) \qquad h(x)$	
0	-2 3 4	# Let $w(x) = e^{h(x)} + 5(f(x))^2$ . Find $w(3)$ .
1	$3 \sqrt{2} 2$	
2	$0 - 3 \frac{3}{2}$	
3	$-1$ $\frac{\pi}{2}$ 0	
4	$\frac{4}{6}$ $-\frac{4}{7}$ $-\frac{\pi}{2}$	
5		
Answer:	$\frac{\pi}{4}$	Answer: 0
	g(x)+5 (1) (2)	$\# \_ f(2) + g(3) + h(0)$
#	If $p(x) = \frac{f(x)}{f(x)-6}$ , find $p(3)$ .	
Answer:	5	Answer: 6
#	Let $h^{-1}(x)$ be defined as the inverse of	# Let $r(r) = \sqrt{7 - f(r)}$ Find $r(0)$
	$h(x)$ . Find $h^{-1}(2)$ .	$\pi$ Let $f(x) = \sqrt{f(x)}$ . Find $f(0)$ .
Answer:	$4\sqrt{2}$	Answer: $\frac{\pi+20}{-28}$
<u>+</u>	If $n(x) = h(x) - q(x)$ find $n(1)$	
#	n p(x) = n(x) - g(x), mu $p(1)$ .	# Find the average rate of change of $h(x)$
		on the closed interval [0, 4].
Answer:	$-\frac{\pi}{2}-1$	Answer: 1
	δ	$a(x) + \overline{b}$
#	$\operatorname{Arcsin}(f(3)) + \operatorname{Arcsec}(g(1))$	# For what x – value is $p(x) = \frac{g(x)+5}{f(x)-6}$
		undefined?

Directions: Beginning in cell #1, read the question and show the work necessary to answer it (attach separate sheets if necessary). Search for your answer and call that cell #2. Continue in this manner until you complete the circuit. Note: The last question will not have a match!

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# 1 Find the slope of the line which connects the point (b, 3b) to the point (3b, 6b). [Note: b≠0.]	Answer: $\frac{-1+ln3}{2}$ # The graph of y = 2 sin(3x - $\frac{\pi}{2}$ ) has an amplitude of, a period of, and a phase shift of to the (left/right) when compared to the graph of y = sinx.
Answer: $\frac{2e}{1-e}$	Answer: 4/5
# As x grows infinitely large, the value of	# Find the average rate of change of
$h(x) = \frac{2x}{5x+8}$ approaches what number?	$w(x) = 3x^2 + 1$ over the interval [-1, 4].
Answer: 75 # For $\frac{\pi}{2} \le A \le \pi$ , $sinA = \frac{3}{5}$ . Find $sin(2A)$ .	Answer: 9 # If $f(x) = lnx$ and $g(x) = e^{x+1}$ , find f(g(2)) - g(f(e)).
Answer: 21	Answer: $(-\infty, 2) \cup (2, \infty)$
# $f(x) = g^{-1}(x)$ and $g(x) = \frac{2x}{x-1}$ ; $f(5) = ?$	# $log_{10}25 + log_{10}4 =$
Answer: [-2, 2]	Answer: $x = -3$
# Solve for x: $e^{2x+1} - 3 = 0$	# State the domain of $y = \ln(x - 2)$ .
Answer: $2/5$ # The expression $3x^2$ is used to calculate the slope at any point on the graph of the function $g(x) = x^3 - 1$ . Write the equation of the line tangent to $g(x)$ at its x-intercept.	Answer: 3/2 # The linear function f(x) is parallel to the line $y = \frac{4}{5}x - 7$ and passes through the point (-5, 0). What is f(-6)?

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Answer: -4/5	Answer: 2
# The quadratic function g(x) has a vertex at (-5, 0) and y-intercept of (0, -5). What is g(1)?	# The graph of $g(x) = -\sqrt{4 - x^2}$ is a semicircle in quadrants III and IV. Find the domain of $g(x)$ .
Answer: 4 # Simplify the expression $\frac{x^3+125}{x+5}$ and then evaluate the resulting expression for x = -5.	Answer: 26 # Find $x^2 - y^2$ given that x + y = 7 and x - y = 3.
Answer: $3 - e^2$ # Given $f(x) = x^2 + 5$ , find $\frac{f(3+h)-f(3)}{h}$ $(h \neq 0)$ .	Answer: 36 # State the range of $w(x) = \frac{2x+1}{x+3}$ .
Answer: x > 2 # $81^{\frac{3}{4}} + 8^{\frac{2}{3}} + 125^{\frac{1}{3}}$	Answer: -24/25 # The graphs of $g(x) = \ln(x + 3)$ and $f(x) = \frac{2x+1}{x+3}$ have the same vertical asymptote. What is it?
Answer: 5/3 # Solve for x: $\ln(x) - \ln(x + 2) = 1$	Answer: $y = 3x - 3$ # Evaluate g(x) = 5sinx + cos(2x) for x = $\frac{\pi}{2}$ .
Answer: -36/5 # Find the average rate of change of the function $p(x) = \frac{4}{5}x - 2$ from x=0 to x=15.	Answer: 6 + h # If the perimeter of a rectangle is 68 and the width is 10, find the length of a diagonal.

Name \_\_\_\_\_

Beginning in cell #1, use a combination of analytic methods and a graphing calculator to solve the problem. Show how you arrived at your answer, even if a lot of your work was done on the calculator. Hunt for your answer and call this problem #2. Continue in this manner until you complete the circuit. Note: Answers are rounded or truncated to three decimal places. Also, make sure you know HOW to do these on the test when there are no answer choices!

Answer:	0.510	Answer:	1.771
#1 Find ƒ(λ	the average rate of change for the function $f(x) = 3e^{-x}$ from $x = -1$ to $= 7$ .	#	The function $r(x) = \frac{x+2}{2x-3}$ has a horizontal asymptote of y =
Answer:	-1 750	Answer <sup>.</sup>	5 832
#	Find $f(g\left(-\frac{4\pi}{7}\right))$ if $f(x) = \begin{cases}  x  - 2, \ x \le 0\\ \frac{3}{x}, \ x > 0 \end{cases}$ and $g(x) = \tan x$ .	#	Find the zero of $f(x) = 3 - 2^x$ .
Answer: #	1.585 Suppose the number of cases of a rare disease is able to be reduced by 25% annually. If there are 4000 cases nationwide, how many years will it take to reduce the number of cases to 300 ?	Answer: #	1.500 The graph of an exponential function, $y = a \cdot b^x$ , passes through the points (1, 1) and (2, 3.5). Find the value of $a$ .
Answer: #	0.500 If $f(g(x)) = g(f(x)) = x$ , and $g(x) = 2 + \ln(x + 1)$ , find $f(4)$ .	Answer: #	9.899 A cone has a height which is one-sixth the radius. If the radius is two, what is the volume of the cone?
Answer: #	1.396 $g(x) = \ln(x - 4)$ and $f(x) = \frac{1}{2}x^2 + 3$ . Find $f(g(6))$ .	Answer: #	0.685 A drug is administered intravenously for eight hours, $0 \le t \le 8$ , and the function $f(t) = 32 - 8.2\ln(1 + 2t)$ gives the number of units of the drug in the body after t hours. How many units are present after 7 hours (at time $t = 7$ )?

Answer:	9.004	Answer:	-1.019
#	What is the period of $y = \sin(4x)$ ?	#	For $g(x) = -3x^2 + 5.2x + 7$ , find the maximum value of the function.
Answer	1.760	Answer	0.456
#	Solve for $\theta$ , $\frac{3\pi}{2} \le \theta \le 2\pi$ . $\cos\theta = 0.9$	#	What is the minimum value of $y = -3\cos t + 1.25$ ?
Answer:	9.794	Answer:	3.240
#	The function $v(t) = -9.8t + 5$ gives the instantaneous velocity (in m/sec) of an object thrown upward with an initial velocity of 5 m/sec. At what time t does the object start falling?	#	Solve the non-linear system $\begin{cases} y = \sqrt{x+2} \\ y = 1.1x^5 \end{cases}$ . To advance in the circuit, locate the y-coordinate of the solution.
Answer:	9.253	Answer:	6.389
#	An isosceles right triangle has a leg of 7 cm. What is the length of the hypotenuse, in cm?	#	Solve $\sec(3x) = 5$ on the open interval $\left(0, \frac{\pi}{6}\right)$ .
Answer:	0.286	Answer:	1.571
#	log <sub>3</sub> 7 =?	#	The function $f(x) = \frac{x+2}{2x-3}$ has a vertical asymptote at x =