Precalculus Plus
Summer Assignment 2024 **Name** ______________________________

Part I: Graphing Parent Functions and the Rule of Four

Quick Review – The Rule of Four

There are four basic ways of representing functions: **verbal, algebraic, numerical, and graphical**.

- Verbal includes listing the name of the function and describing its characteristics.
- Algebraic includes writing the equation of the parent function.
- Numerical includes writing 5 key anchor points and features, such as the location of any hole(s) and the equations for all asymptotes (if any). Choose "nice" x values – often we use x values of -2, -1, 0, 1, 2 unless the function is not defined for those values or those values are hard to compute.
- Graphical includes plotting points and sketching the graph. Remember to indicate your scale on both axes.

Example: Absolute Value Parent Function

Verbal:
Name: *Absolute Value Function*

Domain: $(-\infty, \infty)$ Range: $[0, \infty)$

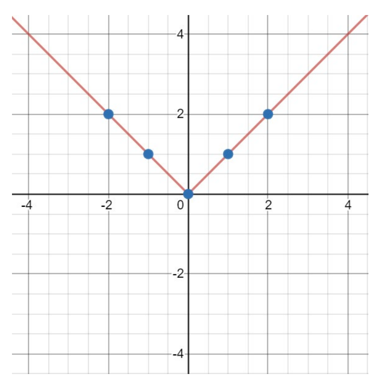
Algebraic (Formula):

$$y = |x| = \begin{cases} x & if \quad x \geq 0 \\ -x & if \quad x < 0 \end{cases}$$

Numerical (Anchor Points):

$(-2, 2)$
$(-1, 1)$
$(0, 0)$
$(1, 1)$
$(2, 2)$

Graphical:



Problem Set I: Graphing Parent Functions and The Rule of Four

Given the name of the following parent functions, complete the "Rule of Four" charts for each.

1. Quadratic Function

Verbal:

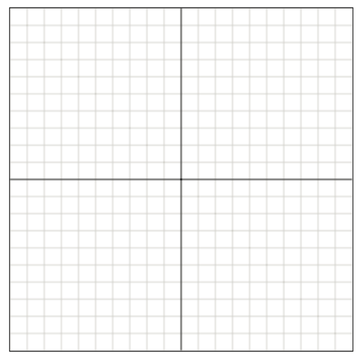
Name: *Quadratic Function*

Domain: Range:

Algebraic (Formula):

Numerical (Anchor Points):

Graphical:



2. Square Root Function

Verbal:

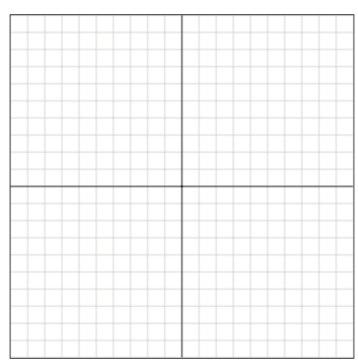
Name: *Square Root Function*

Domain: Range:

Algebraic (Formula):

Numerical (Anchor Points):

Graphical:



3. Cubic Function

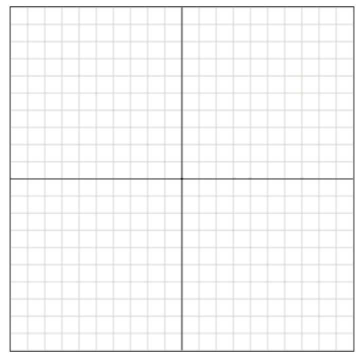
Verbal:
Name: *Cubic Function*

Domain: Range:

Algebraic (Formula):

Numerical (Anchor Points):

Graphical:



4. Cube Root Function

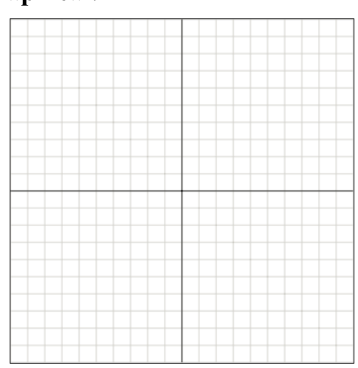
Verbal:
Name: *Cube Root Function*

Domain: Range:

Algebraic (Formula):

Numerical (Anchor Points):

Graphical:



5. Reciprocal Function

Verbal:

Name: *Reciprocal Function*

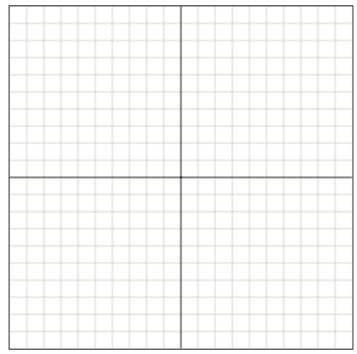
Domain: Range:

Algebraic (Formula):

$$y = \frac{1}{x}$$

Numerical (Anchor Points and Features):

Graphical:



6. Reciprocal Square Function

Verbal:

Name: *Reciprocal Square Function*

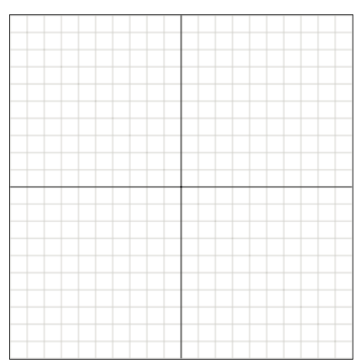
Domain: Range:

Algebraic (Formula):

$$y = \frac{1}{x^2}$$

Numerical (Anchor Points and Features):

Graphical:



Part II: Identifying Characteristics of Functions

Quick Review **– Proper Notation**

When we refer to a portion of a function, we describe the ***x*-values** for that portion using either Set or Interval Notation:

Set Notation: $\{x | x \geq 3\}$ "The set of all x values such that x is greater than or equal to 3."

Interval Notation: $[3, \infty)$ "The set of all real values of x from 3 to infinity, including 3."

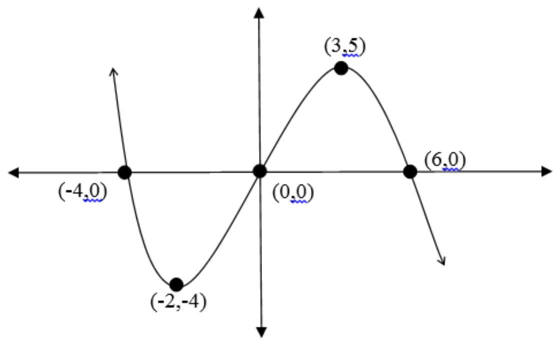
The brackets and parentheses with Interval Notation are very important, for they tell us whether or not the endpoints of the interval are included in the set.

-] and [mean that the endpoint is included
-) and (mean that the endpoint is not included -- **always use a parenthesis with an infinity symbol**
- An interval can be closed on one end and open on the other.
- $\cup$ is the union symbol and means to include all values from the first interval and all values from the second interval.
- $\cap$ is the intersection symbol and means to include only the values that are in **both** the first interval and the second interval.

Quick Review **– Definitions**

- A function f is **increasing** on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.
- A function f is **decreasing** on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.
- $f(c)$ is called a **relative (or local) minimum** of the function f, if there is an open interval I containing c on which $f(c) \leq f(x)$ for all x in I.
- $f(c)$ is called a **relative (or local) maximum** of the function f, if there is an open interval I containing c on which $f(c) \geq f(x)$ for all x in I.
- $f(c)$ is called an **absolute (or global) minimum** of the function f if $f(c) \leq f(x)$ for all x in the domain of f.
- $f(c)$ is called an **absolute (or global) maximum** of the function f if $f(c) \geq f(x)$ for all x in the domain of f.
- A smooth, continuous function f is **concave up** at c in its domain if the graph of f lies above the tangent line to f at c.
- A smooth, continuous function f is **concave down** at c in its domain if the graph of f lies below the tangent line to f at c.
- Generally speaking, a function is **continuous** if it does not contain any holes, jumps, or vertical asymptotes.
- A continuous function is described as **smooth** if it does not contain any corners or cusps.

Example: Identifying Characteristics of a Function



Characteristics

Increasing: $[-2,3]$

Decreasing: $(-\infty,-2]\cup[3,\infty)$

Constant: Never

Global/Absolute Maximum: None
Global/Absolute Minimum: None

Local/Relative Maximum: point $(3,5)$
A local maximum value of 5 occurs at x = 3.
Local/Relative Minimum: point $(-2,-4)$
A local minimum value of – 4 occurs at x = –2.

Positive: $(-\infty,-4)\cup(0,6)$

Negative: $(-4,0)\cup(6,\infty)$

Zero: $\{x|x=-4,0,6\}$

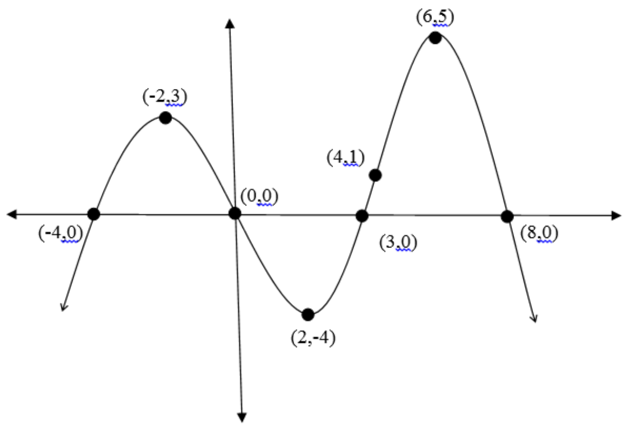
Concave Up: $(-\infty,0)$

Concave Down: $(0,\infty)$

Points of Inflection (point at which concavity changes): $(0,0)$

End Behavior: $f(x)\to-\infty$ as $x\to\infty$
$f(x)\to\infty$ as $x\to-\infty$

Problem Set II: Identifying Characteristics of Functions



Fill in the characteristics for the function above using proper notation. Assume (0, 0) and (4, 1) are the points of inflections (points at which concavity changes).

Characteristics
Increasing:
Decreasing:
Constant:
Global/Absolute Maximum:
Global/Absolute Minimum:
Local/Relative Maximum:
Local/Relative Minimum:
Positive:
Negative:
Zero:
Concave Up:
Concave Down:
Points of Inflection: (0,0) and (4,1)
End Behavior:

<u>Part III: Solving Equations and Inequalities</u>

Students should be able to solve linear, quadratic, polynomial, rational, radical, and absolute value equations and inequalities.

Problem Set III-A: Solve the following equations. You should NOT use a calculator to solve. <u>All solutions should be given as an exact answer (preferably simplified) – **NO decimal approximations.**</u>

1. $\left[6-4x+2(x-7)\right]-52-3(2x-4)=6\left[3(2x-3)+6\right]$

2. $8-3\left|\frac{1}{2}b-4\right|=2$

3. $-2(2x-3)^2+14=0$

4. $4t^3+4t^2-2t=0$

5. $\frac{x}{x+2}-4=\frac{x+1}{x}$

6. $\frac{2x}{x-1}+\frac{1}{x-3}=\frac{2}{x^2-4x+3}$

7. $2\sqrt{4-y}+10=12$

8. $\sqrt{x+2}=6-\sqrt{7x+2}$

9. $x^{2/3}+3x^{1/3}+2=0$

10. $p(2p-5)^2-3(2p-5)=0$

Problem Set III-B: Solve the following inequalities. Show all work including any sign chart analysis.

11. $-6 \le 1 - 4(x+2) \le 16$

12. $|1-2x| < 4$

13. $|2-5x| > 0$

14. $6x^2 - 7x < 20$

15. $\dfrac{3}{x-2} - \dfrac{1}{x-4} \le 0$

Part IV: Function Transformations

Quick Review **– Graphing with the Transformation Approach**

There are three types of transformations:

- horizontal/vertical **reflections** (flip over *x*-axis or *y*-axis)
- horizontal/vertical **scaling** (stretching and shrinking)
- horizontal/vertical **translations** (shifts up/down or left/right).

Transformation Rule $(x_o, y_o) \to (x_n, y_n)$ describes how to take a parent equation/graph anchor point and transform it to the corresponding "sibling" anchor point.

Example: Graphing a Transformed Equation and Writing a Transformation Rule

Graph: $y = \frac{1}{3}(-2x+8)^3 - 5$

a. Decide what ***parent*** function is in the equation, $y = \frac{1}{3}(-2x+8)^{\mathbf{3}} - 5$.

Parent Function: $y = x^3$

b. Find the ***horizontal*** transformations by solving the expression *inside* the parent function for x_n.

$y = \frac{1}{3}(\mathbf{-2x+8})^3 - 5 \longrightarrow x_o = -2x_n + 8$

$x_n = -\frac{1}{2}x_o + 4$

Find the ***vertical*** transformations by looking at the *outside* of the parent function.

$y = \frac{1}{3}(-2x+8)^3 - 5 \longrightarrow y_n = \frac{1}{3}y_o - 5$

Write the Transformation Rule: $(x_o, y_o) \to \left(-\frac{1}{2}x_o + 4,\ \frac{1}{3}y_o - 5\right)$

c. Verbally describe the transformations. You can determine these from the rule. Make sure to think about the order of operations!

- Horizontal flip (reflection) over the *y*-axis
- Horizontal shrink by a factor of $\frac{1}{2}$
- Horizontal Shift Right 4 units
- Vertical shrink by a factor of $\frac{1}{3}$
- Vertical Shift Down 5 units

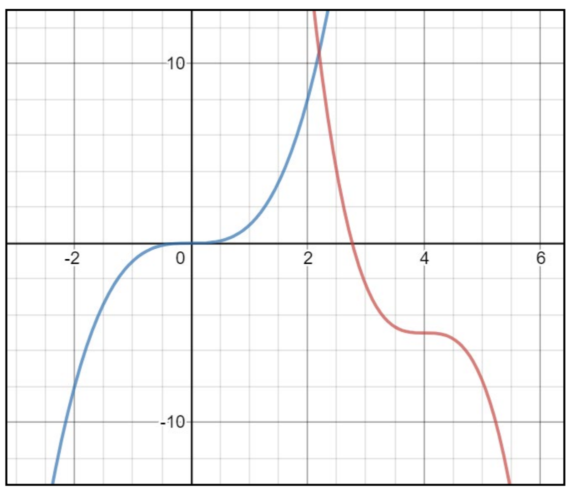
d. Make a list of Anchor Points on the Parent Function (include any critical points on the graph, i.e. vertices, asymptotes, points of inflection). Take each point and substitute the *x* and *y* coordinates into the Anchor Point Rule:

$(x_o, y_o) \rightarrow (x_n, y_n)$

x_o	$y_o = (x_o)^3$	→	$-\frac{1}{2}x_o + 4$	$\frac{1}{3}y_o - 5$
-2	-8		5	$-7.\overline{6}$
-1	-1		4.5	$-5.\overline{3}$
0	0	→	4	-5
1	1		3.5	$-4.\overline{6}$
2	8		3	$-2.\overline{3}$

Notice the old point of inflection (0, 0) has moved to (4, -5), the new point of inflection.

e. Sketch both graphs and note how the parent graph has been changed. Use different colors for the two graphs. Show all the old and new anchor points. (Here I am using blue for the parent graph and red for the transformed graph.)



To: 9th Grade Precalculus Plus Students

You may be unfamiliar with the transformation theory and notation that we teach at MLWGS. Ms. Hamilton (another teacher at MLWGS) has created a very nice and thorough 30-minute video on transformations that you might find helpful to watch **after** reading the previous example and **before** doing the following problem set. https://youtu.be/zxjgWwkJzGQ

Problem Set IV: Function Transformations

Find the transformation rule given the transformed equation:

1. $y=\sqrt{2x-3}+1$ $\qquad (x_0\ ,\ y_o)\rightarrow($ ______________ , ________________ $)$

parent ___________

2. $y=\frac{2}{3}(4x+1)^3-3$ $\qquad (x_0\ ,\ y_o)\rightarrow($ ______________ , ________________ $)$

parent ___________

Find the transformed equation given the transformation rule and parent:

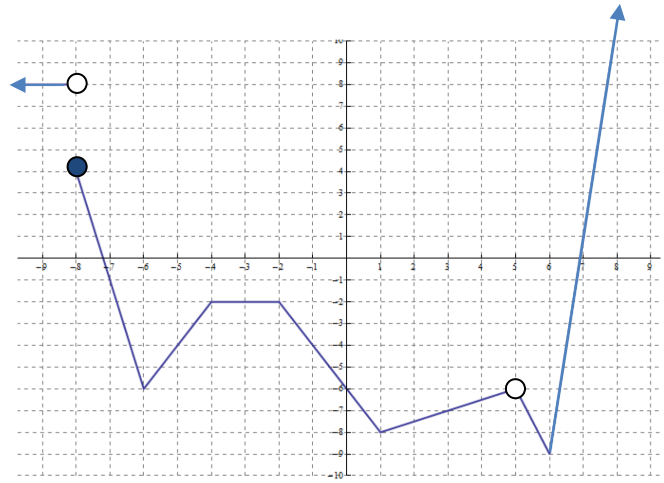
3. $(x_o,y_o)\rightarrow\left(x_n=-2x_o+3,\ y_n=\frac{1}{2}y_o+4\right)$ with parent $y=x^3$

4. $(x_o,y_o)\rightarrow\left(x_n=\frac{2}{3}x_o-\frac{4}{9},\ y_n=-3y_o-2\right)$ with parent $y=\sqrt{x}$

5. $(x_o,y_o)\rightarrow(x_n=4x_o-1,\ y_n=3y_o+2)$ with parent $y=\frac{1}{x^2}$

6. Graph the indicated transformation given the function $f(x)$ shown below.

$$y = \frac{1}{2} f(3x-1) - 2$$



Transformation rule:

$(x_o, y_o) \rightarrow (x_n =$ ________, $y_n =$ ________$)$

Numerical (Anchor Points):

7. State the parent function and the transformation rule, describe the transformations verbally, find anchor points for both, and then graph both the parent and the transformed equation on the same plane.

$$y = -|2x-1| + 3$$

Parent Function: ________________

$(x_0\ ,\ y_o) \rightarrow ($ ______________ , ________________$)$

Verbal Description of Transformations:

Anchor Points: