BC Calculus Summer Assignment (HAMILTON)

This summer assignment is for students who have already completed the AP Calculus AB course at MLWGS and are enrolled in the follow up course BC calculus only. **(If you are enrolled in AP Calculus AB/BC, please see the summer assignment specifically for that course from Ms. Reed).**

Success in BC Calculus will require strong familiarity with topics covered in the AP Calculus AB curriculum.

Required Summer Work: **Complete all reviews for UNITS 6, 7, and 8** in the handout BC Calculus Summer Assignment BIG REVIEW Units 1 – 8. These reviews for Unit 6, 7 and 8 will be collected on the first day of class. Students are expected to complete each review with their own attempt and should then check and correct their answers using the keys provided in a different color pen. You may watch videos on topics that you need more review on at Flipped Math https://calculus.flippedmath.com/version-1.html

Optional Summer Work: While the **summer assignment review for Units 1 – 5 are optional** you should be familiar and able to complete these types of problems as well. **You will not be required to turn in work for units 1 – 5 reviews.** You will be expected to hit the ground at beginning of the year with a strong foundation in Units 1 – 8 but we will be focusing on Unit 6 and new integration techniques at the beginning of the year and quickly applying these new techniques to units 6, 7 and 8 applications.

Looking forward to working with you again next year.

Ms. Hamilton

REQUIRED REVIEW
UNITS 6 - 8

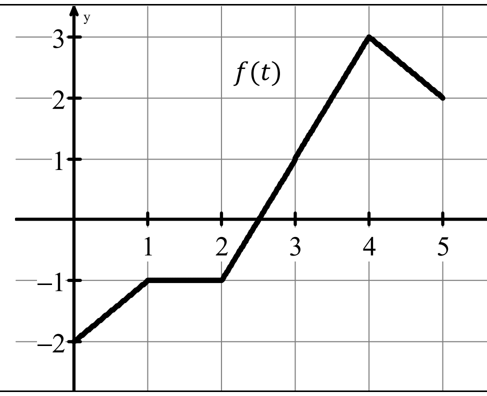
Name: ______________________________ Date:______________ Period: _____ **Review**

Mid-Unit 6 Review – Integration and Accumulation of Change

Lessons 6.1 through 6.5

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 6.

1. Let $g(x) = \int_a^x f(t)\,dt$ with the graph of f shown above and a is a constant. Find the x-values of g regarding each of the following conditions.



a. Relative minimum(s)

b. Relative maximum(s)

c. Concave up

d. Concave down

e. Increasing

f. Decreasing

g. Point(s) of inflection

h. Given $h(x) = \int_0^{\frac{x}{4}+1} f(t)\,dt$. Find the x-value where h has a relative minimum.

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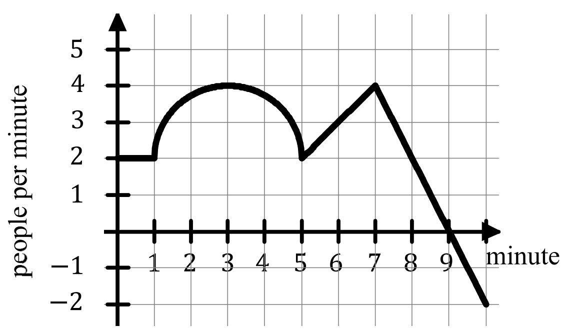
2. The graph below shows the rate of change of the number of people in line for a concert.



a. How many people has the line gained or lost after 5 minutes? Round or truncate to 3 decimal places.

b. How many people has the line gained or lost after 10 minutes? Round or truncate to 3 decimal places.

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3. The graph shows which of the following?

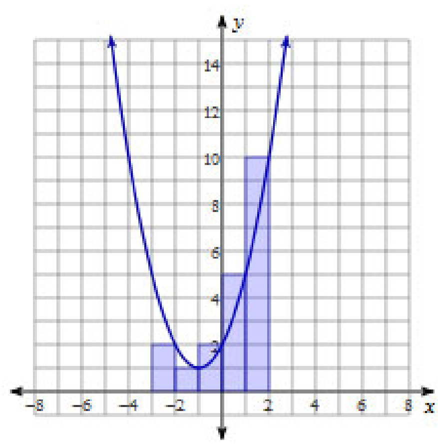
(A) Left Riemann Sum with 5 subintervals

(B) Right Riemann Sum with 5 subintervals

(C) Midpoint Riemann Sum with 5 subintervals

(D) Trapezoidal Approximation with 5 subintervals

(E) None of the above



4. Use a **Left**-Riemann sum with 4 subintervals to approximate the integral based of the values in the table.

$$\int_{0}^{10} f(x)\,dx$$

x	0	4	6	7	10
$f(x)$	3	2	4	5	7

5. Use a Trapezoidal approximation with 4 subintervals to approximate the area under $f(x) = \frac{1}{4}x^2 - 2x + 6$ on $[-3,0]$

6. Write a definite integral that is equivalent to $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{3}{n}\right)\left(-2 + \frac{3k}{n}\right)^4$. The lower limit for the integral is -2.

Find $F'(x)$.	
7. $F(x) = \int_0^{\cos x} t^2\,dt$	8. $F(x) = \int_{x^2}^{8-x} (2t+5)\,dt$

Name: _______________________________ Date:_______________ Period: _____ **Review**

End-of-Unit 6 Review – Integration and Accumulation of Change

Lessons 6.6 through 6.14

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 6.

Find the value of the definite integral.

1. $\int_{-2}^{-1}\left(\frac{1}{x^2}+x^2-5x\right)dx$

2. $\int_{-1}^{8}\left(x^{2/3}-x\right)dx$

3. $\int_{0}^{\pi}(x-\sin x)\,dx$

4. $\int_{-1}^{1}x\sqrt{1-x^2}\,dx$

5. $\int_{0}^{\frac{\pi}{6}}\frac{\sin(2x)}{\cos^2(2x)}\,dx$

6. $\int_{e}^{e^2}\frac{1}{x\ln x}\,dx$

7. If $\int_{-5}^{2}f(x)dx=-17$ and $\int_{5}^{2}f(x)dx=4$, what is the value of $\int_{-5}^{5}f(x)dx$?

(A) -21 (B) -13 (C) 0 (D) 13 (E) 21

Find the following indefinite integrals.

8. $\int \left(\frac{x^2-x+5}{x}\right) dx$

9. $\int \sec x \tan x \, dx$

10. $\int (e^x + 2^x) \, dx$

11. $\int \left(\frac{1}{x}+\frac{1}{x^3}\right) dx$

12. $\int \sqrt{x}\left(x-\frac{4}{x}\right) dx$

13. $\int \frac{50x^3-55x^2-26x+33}{10x-7} dx$

14. $\int \frac{1}{x^2+2x+2} dx$

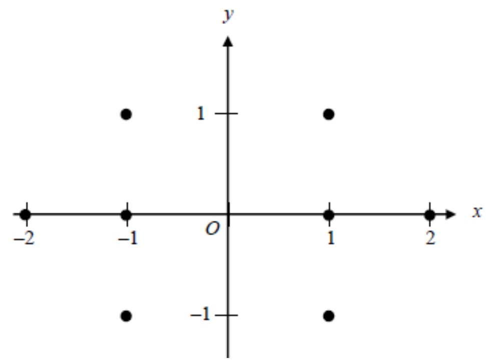
15. **Calculator active problem.** If $f'(x) = \sin(e^x)$ and $f(0) = 5.7$, then $f(2) =$

Name: _______________________________ Date:_______________ Period: _____ **Review**

Unit 7 Review – Differential Equations

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 7.

1. Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x}$, where $x \neq 0$.



   a. On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.

   b. Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = -1$.

   c. Write an equation for the tangent line to the curve $y = f(x)$ through the point $(1, -1)$. Then use your tangent line equation to estimate the value of $f(1.2)$.

2. The rate of change of the volume, $V(t)$, of water in a swimming pool is directly proportional to the cube root of the volume. If $V = 27\text{ ft}^3$ when $\frac{dV}{dt} = 5$, what is a differential equation that models this situation?

Find the general solution of the differential equation.	
3. $\frac{dy}{dx} = \frac{2x}{y}$	4. $\frac{dy}{dx} = x(y + 4)$

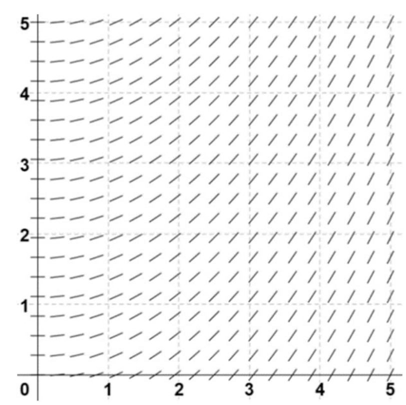
For each differential equation, find the particular solution that passes through the given point.

5. $\frac{dy}{dx} = \frac{18}{6x+3} + \frac{4}{x^3}$; $\left(-\frac{1}{3}, -15\right)$

6. $\frac{dy}{dx} = 2y$ and $y = -0.2$ when $x = 0$

7. A population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 12 years, then what is the value of k?

8. Explain why the following slope field cannot represent the differential equation $\frac{dy}{dt} = 0.4y$



9. For what value of k, if any, will $y = k\cos(2x) + 3\sin(4x)$ be a solution to the differential equation $y'' + 16y = -6\cos(2x)$?

Name: ______________________________ Date:______________ Period: _____ **Review**

Mid-Unit 8 Review – Applications of Integration

Lessons 8.1 through 8.6

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 8.

Average Rate of Change	Mean Value Theorem	Average Value of a Function
$\dfrac{f(b)-f(a)}{b-a}$	$f'(c)=\dfrac{f(b)-f(a)}{b-a}$	$\frac{1}{b-a}\int_a^b f(x)\,dx$

Find the average value of each function on the given interval.

1. $f(x)=x^3$ on $[0,2]$

2. $f(x)=\frac{1}{x}$ on $[1,e]$

$\int$ rate of change =

$\int$ velocity =

$\int$ |velocity| =

3. A particle's velocity is given by $v(t)=6t^2-18t+12$, where t is measured in seconds, v is measured in feet per second, and $s(t)$ represents the particle's position.
   (a) If $s(1)=3$, what is the value of $s(2)$?

   (b) What is the net change in distance over the first 3 seconds?

   (c) What is the total distance traveled by the particle during the first 2 seconds? Show the set up AND your answer.

4. A particle moves along a coordinate line. Its acceleration function is $a(t) = 6t - 22$ for $t \geq 0$. If $v(0) = 24$ find the velocity at $t = 4$.

5. A particle's velocity is given by $v(t) = \cos t$, where t is measured in months, v is measured in kilometers per month, and $s(t)$ represents the particle's position.

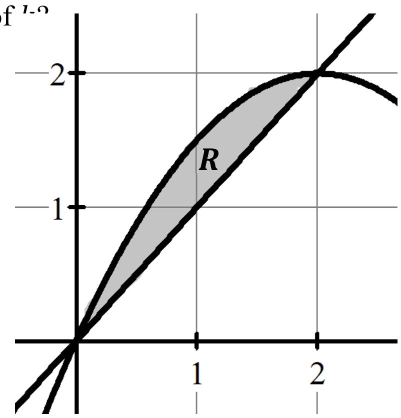
   (a) If $s\left(\frac{\pi}{6}\right) = 10$, what is the value of $s\left(\frac{3\pi}{2}\right)$?

   (b) What is the net change in distance over the first π months?

   (c) What is the total distance traveled by the particle during the first π months? Show the set up AND your answer.

6. Find the area between the two curves $y = x^2 - 4$ and $y = 2 - x$.

7. **Calculator active.** Let R be the region bounded by the graphs $y = 2x - \frac{1}{2}x^2$ and $y = x$ as sown in the figure. If the line $x = k$ divides R into two regions of equal area, what is the value of k?

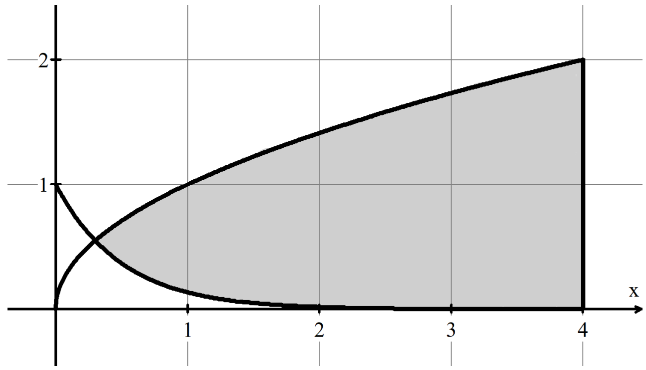


Name: ________________________________ Date:_______________ Period: _____ **Review**

End-of-Unit 8 Review – Applications of Integration

Lessons 8.7 through 8.12

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 8.



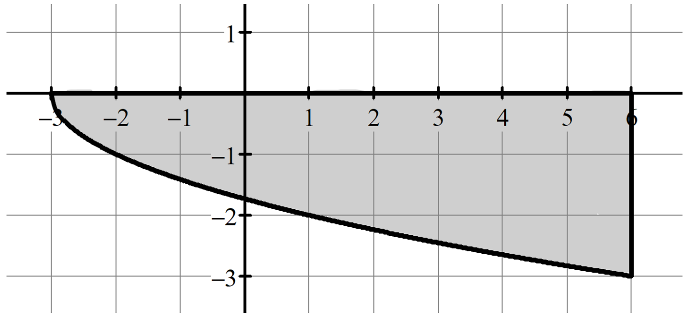
Calculator active. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-2x}$ and the vertical line $x = 4$ as shown in the figure above.

1. Find the area of R

2. Find the volume of the solid generated when R is revolved about the horizontal line $y = 2$.

3. The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 4 times the length of its base in region R. Find the volume of this solid.

Calculator active. Let T be the region enclosed by the graph of $y = -\sqrt{x+3}$, the vertical line $x = 6$, and the x-axis.



4. Find the area of T.

5. The region T is the base of a solid. For this solid, each cross section perpendicular to the y-axis is a semicircle. Find the volume of this solid.

6. Find the volume of the solid generated when T is revolved about the horizontal line $y = -3$.

7. Find the volume of the solid generated when T is revolved about the vertical line $x = 6$.

8. **Calculator active.** A 10,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \frac{400t}{t+2} \text{ for } 0 \leq t \leq 6.$$

a. Find $\int_0^6 r(t)\, dt$

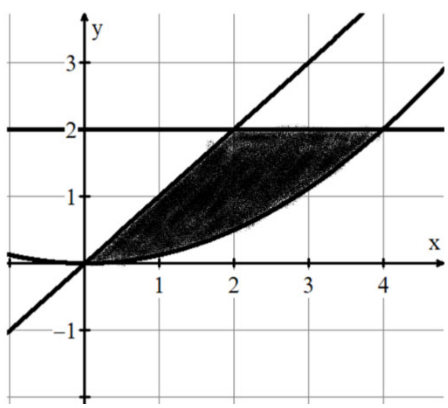
b. Explain the meaning of your answer to part *a* in the context of this problem.

c. Write, but do not solve, an equation involving an integral to find the time A when the amount of water in the tank is 8,000 liters.

Set up the integral(s) that give the area of the region bounded by the given equations. Show the equivalent set up with respect to x as well as with respect to y.

9. $y = x,\ \ y = \frac{x^2}{8},\ \ y = 2$

with respect to x



with respect to y

OPTIONAL REVIEW
UNITS 1 - 5

Name: ______________________________ Date:______________ Period: _____ **Review**

Mid-Unit 1 Review – Limits

Lessons 1.1 through 1.9

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 1.

A salesman tracks the number of cars he sells through the model c, where $c(m)$ is number of cars sold and m is the month for $0 \leq m \leq 24$.		
1. What does $c(10)$ represent?	2. What does $\frac{c(16)-c(8)}{16-8}$ represent?	3. What does $\frac{c(7)-c(6.999)}{7-6.999}$ represent?

Evaluate the limit.		
4. $\lim\limits_{x\to 0} \frac{\sqrt{x+19}-\sqrt{19}}{x}$	5. $\lim\limits_{x\to -3} \frac{x+3}{x^2+2x-3}$	6. $\lim\limits_{x\to 0} \frac{\sin^2(3x)}{\sin^2(5x)}$
7. $\lim\limits_{x\to 2^-} \frac{\lvert x-2\rvert}{x-2}$	8. $\lim\limits_{x\to 10} \frac{x^2-5x-50}{x-10}$	9. $\lim\limits_{x\to 0} \frac{\frac{1}{x+1}-1}{x}$

Name: ______________________________ Date:______________ Period: _____ **Review**

End of Unit 1 Review– Limits and Continuity

Lessons 1.10 through 1.16.

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you should review all packets from Unit 1 (including the Mid-Unit Review).

1. If $f(x) = \frac{x+3}{x^2-2x-15}$, identify the type of each discontinuity and where it is located.

State whether the function is continuous at the given x values. Justify your answers!

2. $f(x) = \begin{cases} \cos(3x), & x < 0 \\ \tan x, & 0 \le x < \frac{\pi}{4} \\ \sin(2x), & x \ge \frac{\pi}{4} \end{cases}$

Continuous at $x = 0$?

Continuous at $x = \frac{\pi}{4}$?

Find the domain of each function.

3. $h(t) = \frac{\sqrt{t+3}}{t-5}$

4. $f(x) = \ln\left(\frac{2}{x-1}\right)$

5. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2+6x+8}{x+2}$ when $x \neq -2$, then $f(-2) =$

6. Let f be the function defined by $f(x) = \begin{cases} \frac{x^2+8x+12}{x+6}, & x \neq -6 \\ b, & x = -6 \end{cases}$. For what value of b is f continuous at $x = -6$?

Evaluate the limit.		
7. $\lim_{x\to\infty} \sin\left(\frac{x+3\pi x^2}{2x^2}\right)$	8. $\lim_{x\to -5^-} \frac{-3}{25-x^2}$	9. $\lim_{x\to\infty} \frac{\sin x}{x}$
10. $\lim_{x\to\infty} \frac{4x^5-2x^2+3}{3x^2+2x^5-x^4}$	11. $\lim_{x\to -1} \frac{x^2+1}{x+1}$	12. $\lim_{x\to\infty} x^5 3^{-x}$

13. Identify all horizontal asymptotes of $f(x) = \frac{\sqrt{16x^6+x^3+5x}}{5x^3-8x}$.

10. If $f(x) = \begin{cases} \sin x, & x < -\pi \\ \tan x & -\pi < x < \frac{\pi}{4} \\ \cos x, & x \geq \frac{\pi}{4} \end{cases}$, find the following:

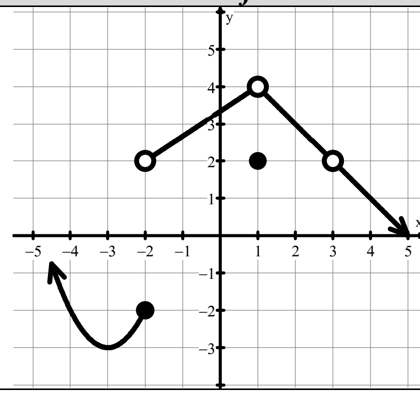
a. $\lim_{x \to -\pi^-} f(x) =$

b. $\lim_{x \to -\pi} f(x) =$

c. $\lim_{x \to \frac{\pi}{4}} f(x) =$

d. $f\left(\frac{\pi}{4}\right) =$

Give the value of each statement. If the value does not exist, write "*does not exist*" or "*undefined*."



11. $\lim_{x \to 3} f(x) =$

12. $\lim_{x \to 1} f(x) =$

13. $f(3) =$

14. $f(-2) =$

15. $\lim_{x \to 2} f(x) =$

16. $\lim_{x \to -2^+} f(x) =$

17. $f(1) =$

18. $\lim_{x \to -2^-} f(x) =$

19. Let g and h be the functions defined by $g(x) = -\frac{1}{4}x^2 - \frac{1}{2}x - \frac{9}{4}$ and $h(x) = \sin\left(\frac{\pi}{2}x\right) - 1$. If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for all x, what is $\lim_{x \to -1} f(x)$?

CALCULATOR ALLOWED:

20. If $f(x) = \frac{x^2+10x+21}{x+3}$, create your own table of values to help you evaluate $\lim_{x \to -3} f(x)$.

$\lim_{x \to -3} f(x) =$

x						
$f(x)$						

Name: ______________________________ Date:______________ Period: _____

Review

Unit 2 Review – Differentiation: Definition & Fundamental Properties

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you should review all packets from Unit 2.

Find the average rate of change of each function on the given interval. Use appropriate units if necessary.

1. $w(x) = \ln x; \quad 1 \le x \le 7$

2. $s(t) = -t^2 - t + 4; \ [1, 5]$
   t represents seconds
   s represents feet

3. Find the derivative of $y = 2x^2 + 3x - 1$ by using the definition of the derivative. $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

4. For the function $h(t)$, h is the temperature of the oven in Fahrenheit, and t is the time measured in minutes.
   a. Explain the meaning of the equation $h(15) = 420$.

   b. Explain the meaning of the equation $h'(43) = -11$.

Find the derivative of each function.

5. $f(x) = 4 - \frac{1}{2x^2}$

6. $g(x) = 3\sqrt{x} - \frac{6}{x^2} + 5\pi^3$

7. $h(x) = 4e^x - 2\cos x$

8. $s(t) = t^2 \sin(t)$

9. $d(t) = 3\sqrt{t} \ln t$

10. $y = \frac{4}{x} - \sec x$

11. $h(x) = \frac{2-x}{x+2}$

Find the equation of the tangent line of the function at the given *x*-value.

12. $f(x) = -2x^3 + 3x$ at $x = -1$.

13. $f(x) = 4\sin x - 2$ at $x = \pi$

14. Find the equation for the normal line of $y = \frac{1}{2}x^2 + \frac{3}{4}x - 4$ at $x = -3$

15. If $f(x) = 3\sin x - 2e^x$ find $f'(0)$. No calculator!

A calculator is allowed on the following problems.

16. If $f(x) = x\sin(3x^2 - 2)$; find $f'(7)$.

17. If $f(x) = \csc(3x)$ at $x = 2$.

18. Use the table below to estimate the value of $d'(120)$. Indicate units of measures.

t seconds	2	13	60	180	500
$d(t)$ feet	10	81	412	808	2,105

FINDING EXTREMA

The First Derivative Test

<table>
<tr><th>STEPS</th><th>EXAMPLE
$\boldsymbol{f(x) = x^2 + 2x + 1}$</th></tr>
<tr><td>1. Find the critical points.</td><td>$f'(x) = 2x + 2$
$0 = 2x + 2$
$x = -1$</td></tr>
<tr><td>2. Determine whether the function is increasing or decreasing on each side of every critical point.
A chart or number line helps!</td><td>

Interval	$(-\infty, -1)$	-1	$(-1, \infty)$
Test Value	-2	-1	2
$f'(x)$	$f'(-2) = -$ Negative	$f'(-1) = 0$	$f'(2) = 6$ Positive

Function decreases to the left and increases to the right of $x = -1$ so it must be relative minimum point</td></tr>
</table>

The Second Derivative Test

STEPS	EXAMPLE $\boldsymbol{f(x) = x^2 + 2x + 1}$
1. Find the critical points.	$f'(x) = 2x + 2$ $0 = 2x + 2$ $x = -1$
2. Determine whether the function is concave up or concave down at every critical point using the second derivative.	$f''(-1) = 2$ Second derivative is positive at $x = -1$ Concave up $x = -1$ is a relative minimum point

Finding Absolute Extrema on an Interval (Candidates Test)

STEPS	EXAMPLE $\boldsymbol{f(x) = x^2 + 2x + 1}$ **on the interval** $\boldsymbol{[-3, 0]}$
1. Find the critical points. The critical points are candidates as well as the endpoints of the interval.	$f'(x) = 2x + 2$ $0 = 2x + 2$ $x = -1$
2. Check all candidates using the $f(x)$.	$f(-3) = 4$ absolute maximum $f(-1) = 0$ absolute minimum $f(0) = 1$

LINEAR MOTION

The chart matches up function vocabulary with linear motion vocabulary.

FUNCTION	LINEAR MOTION
Value of a function at x	Position at time t
First Derivative	Velocity
Second Derivative	Acceleration
$f'(x) > 0$ Increasing Function	Moving right or up
$f'(x) < 0$ Decreasing Function	Moving left or down
$f'(x) = 0$	Not moving
Absolute Max	Farthest right or up
Absolute Min	Farthest left or down
$f'(x)$ changes signs	Object changes direction
$f'(x)$ and $f''(x)$ have same sign	Speeding Up
$f'(x)$ and $f''(x)$ have different signs	Slowing Down

Example:
A particle moves along the x-axis with the position function $x(t) = t^4 - 4t^3 + 2$ where $t > 0$.

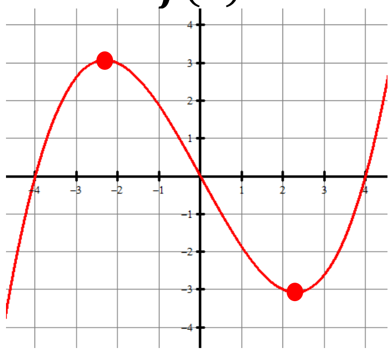
Interval	$(\mathbf{0}, \mathbf{2})$	**2**	$(\mathbf{2}, \mathbf{3})$	**3**	$(\mathbf{3}, \infty)$
$\boldsymbol{x'(t)}$ **velocity**	$x'(t) < 0$ decreasing left	$x'(t) < 0$ decreasing left	$x'(t) < 0$ decreasing left	$x'(t) = 0$ Not moving	$x'(t) > 0$ Increasing right
$\boldsymbol{x''(x)}$ **acceleration**	$x''(t) < 0$ Concave down	$x''(t) = 0$	$x''(t) > 0$ Concave up	$x''(t) > 0$ Concave up	$x''(t) > 0$ Concave up
Conclusion	Speeding Up	Moving Left	Slowing Down	Not Moving	Speeding Up

FUNCTION	LINEAR MOTION
$t = 3$ is minimum	$t = 3$ has no velocity Changing direction
Decreasing (0,3)	Moving left (0,3)
Increasing $(3, \infty)$	Moving right $(3, \infty)$

GRAPHICAL ANALYSIS

Connecting $\boldsymbol{f(x)}$ to $\boldsymbol{f'(x)}$

$\boldsymbol{f(x)}$

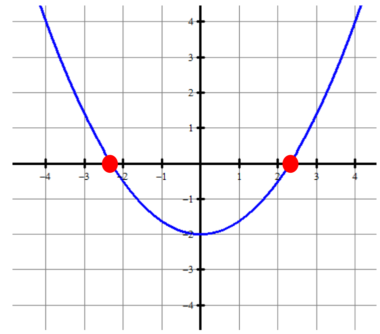


$f(x)$ max at $x = -2.2$ and min $x = 2.2$
so $f'(x) = 0$ at these points

$f(x)$ is increasing on
$(-\infty, -2.2)(2.2, \infty)$
so $f'(x) > 0$ on these intervals

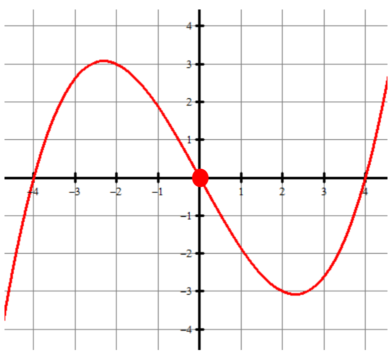
$f(x)$ is decreasing on $(-2.2, 2.2)$
so $f'(x) < 0$ on this interval

$\boldsymbol{f'(x)}$



Connecting $\boldsymbol{f(x)}$ to $\boldsymbol{f''(x)}$

$\boldsymbol{f(x)}$

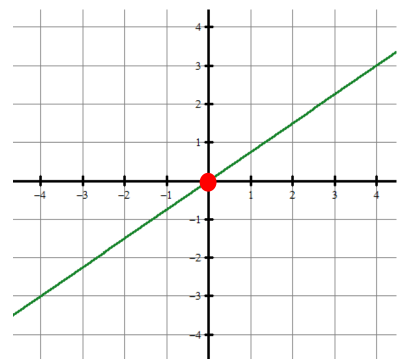


$f(x)$ is concave down on $(-\infty, 0)$
so $f''(x) < 0$ on this interval

$f(x)$ is concave up on $(0, \infty)$
so $f''(x) > 0$ on this interval

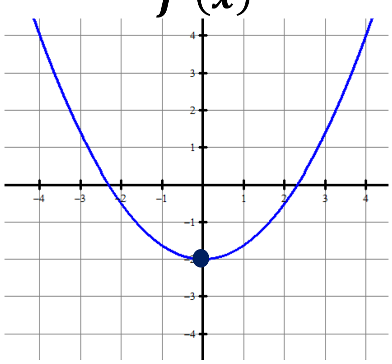
$x = 0$ is a point of inflection on $f(x)$
so $f''(x)$ changes sign at $x = 0$.

$\boldsymbol{f''(x)}$



Connecting $\boldsymbol{f'(x)}$ to $\boldsymbol{f''(x)}$

$\boldsymbol{f'(x)}$

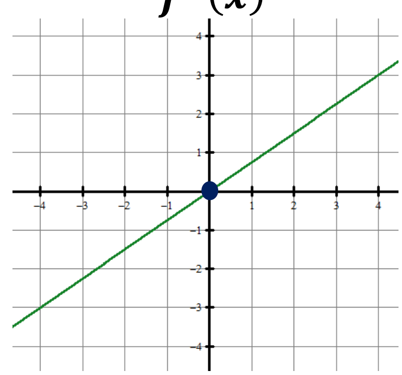


$f'(x)$ min $x = 0$
so $f''(x) = 0$ at $x = 0$

$f'(x)$ is decreasing on $(-\infty, 0)$
so $f''(x) < 0$ on this interval

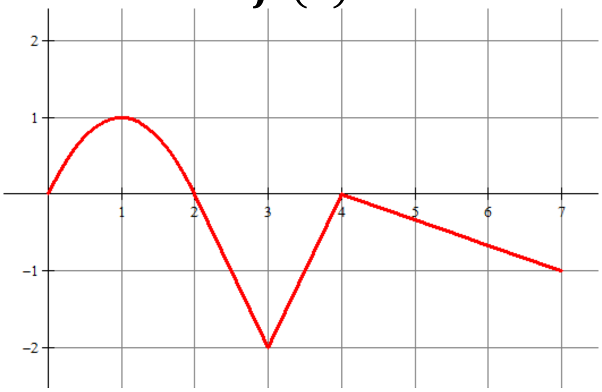
$f'(x)$ is increasing on $(0, \infty)$
so $f''(x) > 0$ on this interval

$\boldsymbol{f''(x)}$



Using $f'(x)$ to draw conclusions about $f(x)$

$f'(x)$



Find Extrema of $f(x)$

$x = 2$ and 4 are critical points because $f'(x) = 0$

$x = 2$ is a maximum because $f'(x)$ is positive on left, negative on right

$x = 4$ is NOT an extrema because $f'(x)$ is negative on left, negative on right

Find Points of Inflection of $f(x)$

$x = 1, 3,$ and 4 are possible points of inflection because $f''(x) = 0$ or DNE

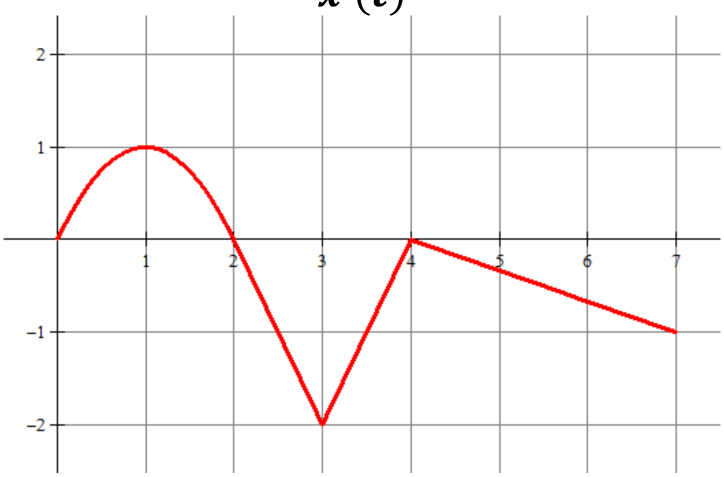
$x = 1$ is a point of inflection because $f''(x)$ changes sign from positive to negative at $x = 1$.

$x = 3$ is a point of inflection because $f''(x)$ changes sign from negative to positive at $x = 3$.

$x = 4$ is a point of inflection because $f''(x)$ changes sign from positive to negative at $x = 4$.

Now interpret the same graph as linear motion if the graph represents velocity of a particle moving along x-axis.

$x'(t)$



Moving Right or Left?

Particle moves right on (0,2).

$t = 2$ particle changes direction.

Particle moves left on (2,4)(4,7).

$t = 4$ particle has no velocity.

The maximum speed happens at $t = 3$.

Speeding up or Slowing down?

Particle speeds up on (0,1) because $f'(x)$ has the same sign as $f''(x)$

Particle slows down on (1,2) because $f'(x)$ has a different sign from $f''(x)$

Particle speeds up on (4,7) because $f'(x)$ has the same sign as $f''(x)$

Particle speeds up on (2,3) because $f'(x)$ has the same sign as $f''(x)$

Particle slows down on (3,4) because $f'(x)$ has a different sign from $f''(x)$

Name: ________________________________ Date:_______________ Period: _____ **Review**

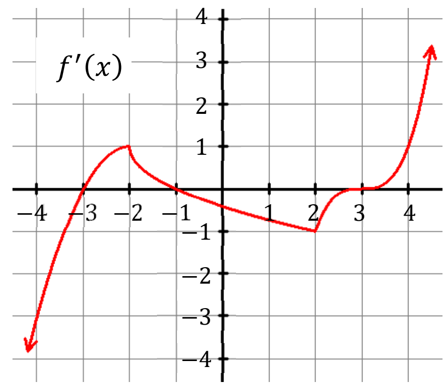
Mid-Unit 5 Review – Analytical Applications of Differentiation

Lessons 5.1 through 5.7

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 5.

1. If $y = -2x^2 + 4x + 3$ apply the Mean Value Theorem to find when the instantaneous rate of change will equal the average rate of change on the interval $[1, 3]$.

2. Below is the graph of f'. Find all relative extrema of f and justify.



3. The derivative of g is given by $g'(x) = 6x^2 - 6$. Find all relative extrema and justify your conclusions.

4. What is the minimum value of $f(x) = xe^{\frac{x}{3}}$?

5. **Calculator active problem.** The derivative of f is defined by $f'(x) = \sin(x - x^2)$ for $0 \leq x \leq 3$. On what interval(s) is f decreasing?

6. What is the absolute maximum value AND the absolute minimum value of the function $g(x) = x^3 - 12x$ on the closed interval $[0, 4]$.

7. Use the 2nd Derivative Test to find x-values of the extrema of $g(x) = 2\cos x - x$ on the interval $(0, 2\pi)$ and justify your answer.

8. Find the intervals of concavity for the function $f(x) = x^4 + 4x^3 - 18x^2 - 4x + 7$

Name: _______________________________ Date:_______________ Period: _____

Review

Unit 4 REVIEW – Contextual Application of Differentiation

Reviews do NOT cover all material from the lessons but should remind you of key points. To be prepared, you should review all packets from Unit 4.

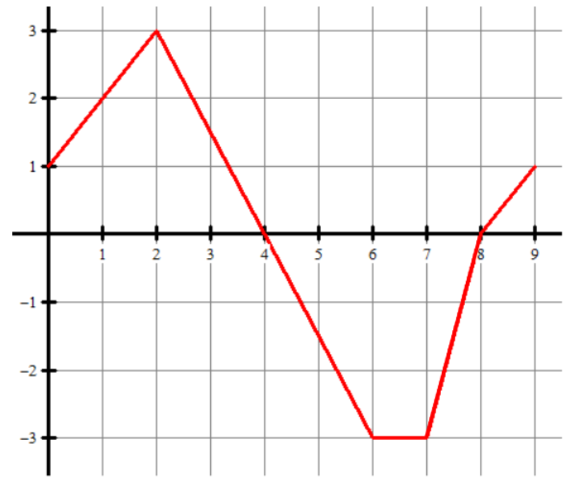
1. The figure shows the velocity $v = \frac{ds}{dt} = f(t)$ of a body moving along a coordinate line in meters per second.

   a) When does the body reverse direction?

   b) When is the body moving at a constant speed?

   c) What is the body's maximum speed?

   d) At what time interval(s) is the body slowing down?



Find the following. Use L'Hospital's when possible.		
2. $\lim_{x \to 2} \frac{x-2}{x^2-7x+10}$	3. $\lim_{x \to 0} \frac{3x^2}{e^x-1-x}$	4. $\frac{d}{dx} \frac{3x-2}{5x+1}$

5. If the length l of a rectangle is decreasing at a rate of 2 inches per minute while its width w is increasing at a rate of 2 inches per minute, which of the following must be true about the area A of the rectangle?

   (A) A is always increasing. (B) A is always decreasing. (C) A is increasing only when $l > w$.

   (D) A is increasing only when $l < w$. (E) A remains constant.

The following problems are calculator active.

6. Brust is riding his bicycle north away from an intersection at a rate of 15 miles per hour. Sully is driving his car towards the intersection from the west at a rate of 30 miles per hour. If Brust is 0.4 miles from the intersection, and Sully is 1 mile from the intersection, at what rate is the distance between the two of them increasing or decreasing?

7. The side of a cube is increasing at a constant rate of 0.2 centimeters per second. In terms of the surface area S, what is the rate of change of the volume of the cube, in cubic centimeters per second?

(A) $0.1S$ (B) $0.2S$ (C) $0.6S$

(D) $0.04S$ (E) $0.008S$

8. The function $f(x) = (1 - \sin x)^2$ is concave up at $x = \frac{\pi}{6}$?

   a. What is the estimate for $f(0.5)$ using the local linear approximation for f at $x = \frac{\pi}{6}$?

   b. Is it an underestimate or overestimate? Explain.

Name: ______________________________ Date:_______________ Period: _____

Review

Unit 3 REVIEW – Composite, Implicit, and Inverse Functions

Reviews do NOT cover all material from the lessons but should remind you of key points. To be prepared, you should review all packets from Unit 3.

Find the derivative.

1. $h(x) = \cos^2(4x)$

2. $y = \ln\sqrt{x+3}$

3. $x^2 + 2y^5 = 10xy$

4. $y = \csc^{-1}(x^3)$

For each problem, let f and g be differentiable functions where $g(x) = f^{-1}(x)$ for all x.

5. $f(6) = -1$, $f(4) = -2$, $f'(6) = 3$, and $f'(4) = 7$. What is the value of $g'(-1)$?

6. Let f be the function defined by $f(x) = x^3 + 3x + 1$. Let $g(x) = f^{-1}(x)$, where $g(-3) = -1$. What is the value of $g'(-3)$?

Find $\frac{d^2y}{dx^2}$ based on the given information.

7. $y = x^5 - e^{4x}$

8. $y = y^2 + x$

9. Find the equation of the tangent line.
$x^2 + 7y^2 = 8y^3$ at $(-6, 2)$

10. If $x = y^2 - \cos x$ find $\frac{d^2y}{dx^2}$ at $(0, -1)$.

19. Is the function differentiable at $x = 2$?

$$f(x) = \begin{cases} 3x - 3x^2 - 5, & x < 2 \\ 7 - 9x, & x \geq 2 \end{cases}$$

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20. What values of a and b would make the function differentiable at $x = 4$?

$$f(x) = \begin{cases} a\sqrt{x} + bx^2 - 1, & x < 4 \\ \dfrac{16}{x} + bx, & x \geq 4 \end{cases}$$