Name: Solutions Date:______________ Period: _____ **Review**

End-of-Unit 8 Review – Applications of Integration

Lessons 8.7 through 8.12

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 8.

Calculator active. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-2x}$ and the vertical line $x = 4$ as shown in the figure above.

$\sqrt{x} = e^{-2x}$ when $x = 0.300542 \searrow a$

1. Find the area of R

$$A = \int_a^4 \left(\sqrt{x} - e^{-2x}\right) dx$$

$$A = 4.9495$$

2. Find the volume of the solid generated when R is revolved about the horizontal line $y = 2$.

$$V = \pi \int_a^4 \left[(2 - e^{-2x})^2 - (2 - \sqrt{x})^2\right] dx$$

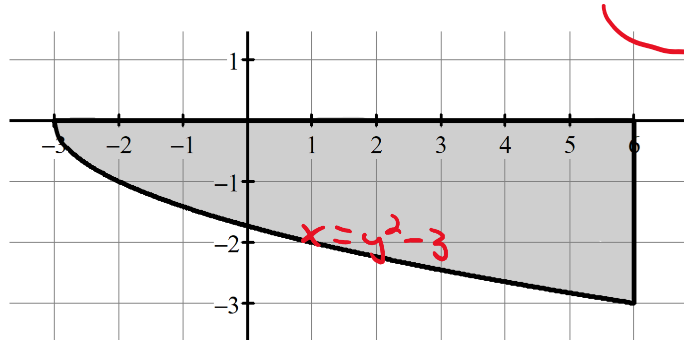
$$V \approx 37.443$$

3. The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 4 times the length of its base in region R. Find the volume of this solid.

$$V = \int_a^4 4\left(\sqrt{x} - e^{-2x}\right)^2 dx$$

$$V \approx 30.2365$$

Calculator active. Let T be the region enclosed by the graph of $y = -\sqrt{x+3}$, the vertical line $x = 6$, and the x-axis.



$-y = \sqrt{x+3}$

$y^2 = x+3$

$x = y^2 - 3$

4. Find the area of T.

$$A = \int_{-3}^{6} \left(0 - -\sqrt{x+3}\right) dx$$

$$A = 18$$

5. The region T is the base of a solid. For this solid, each cross section perpendicular to the y-axis is a semicircle. Find the volume of this solid.

$$V = \int_{-3}^{0} \frac{\pi}{2}\left(\frac{6-(y^2-3)}{2}\right)^2 dy$$

$$V \approx 50.8938$$

6. Find the volume of the solid generated when T is revolved about the horizontal line $y = -3$.

$$V = \pi \int_{-3}^{6} \left[(-3)^2 - \left(-3 - -\sqrt{x+3}\right)^2\right] dx$$

$$V \approx 212.0575$$

7. Find the volume of the solid generated when T is revolved about the vertical line $x = 6$.

$$V = \pi \int_{-3}^{0} \left[(y^2-3-6)^2\right] dy$$

Shift left 6

$$V \approx 407.150$$

Name: Solutions Date:______________ Period: _____ **Review**

Mid-Unit 8 Review – Applications of Integration

Lessons 8.1 through 8.6

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 8.

Average Rate of Change	Mean Value Theorem	Average Value of a Function
$\dfrac{f(b)-f(a)}{b-a}$	$f'(c)=\dfrac{f(b)-f(a)}{b-a}$	$\dfrac{1}{b-a}\int_a^b f(x)\,dx$

Find the average value of each function on the given interval.

1. $f(x)=x^3$ on $[0,2]$

$\frac{1}{2-0}\int_0^2 x^3\,dx$

$\frac{1}{2}\cdot\frac{x^4}{4}\Big|_0^2$

$\frac{1}{8}\left[2^4-0\right]$

$\frac{16}{8}=\boxed{2}$

2. $f(x)=\frac{1}{x}$ on $[1,e]$

$\frac{1}{e-1}\int_1^e \frac{1}{x}\,dx$

$\frac{1}{e-1}\left[\ln x\right]_1^e$

$\frac{1}{e-1}\left[\ln e-\ln 1\right]$

$\boxed{\frac{1}{e-1}}$

$$\int \text{rate of change} = \text{net change}$$

$\int \text{velocity} = \text{displacement}$	$\int \lvert\text{velocity}\rvert = \text{total distance}$

3. A particle's velocity is given by $v(t)=6t^2-18t+12$, where t is measured in seconds, v is measured in feet per second, and $s(t)$ represents the particle's position.

(a) If $s(1)=3$, what is the value of $s(2)$?

$3+\int_1^2(6t^2-18t+12)\,dt$

$3+\left[2t^3-9t^2+12t\right]\Big|_1^2$

$3+(16-36+24)-(2-9+12)=3+4-5=\boxed{2}$

(b) What is the net change in distance over the first 3 seconds?

$\int_0^3(6t^2-18t+12)\,dt$

$\left[2t^3-9t^2+12t\right]\Big|_0^3$

$\left[2(27)-9(9)+36\right]-\left[0\right]=54-81+36=\boxed{9\text{ feet}}$

(c) What is the total distance traveled by the particle during the first 2 seconds? Show the set up AND your answer.

$V(t)=0$

$6(t^2-3t+2)=0$

$(t-2)(t-1)=0$

$t=2 \quad t=1$

$\left|\int_0^1(6t^2-18t+12)\,dt\right|+\left|\int_1^2(6t^2-18t+12)\,dt\right|$

$|5|+|-1|=\boxed{6}$

4. A particle moves along a coordinate line. Its acceleration function is $a(t) = 6t - 22$ for $t \geq 0$. If $v(0) = 24$ find the velocity at $t = 4$.

$v(t) = \int 6t - 22 \, dt$

$v(t) = 3t^2 - 22t + C$

$24 = 0 - 0 + C$

$24 = C$

$v(4) = 3(16) - 22(4) + 24$

$v(4) = 48 - 88 + 24$

$\boxed{v(4) = -16}$

5. A particle's velocity is given by $v(t) = \cos t$, where t is measured in months, v is measured in kilometers per month, and $s(t)$ represents the particle's position.

(a) If $s\left(\frac{\pi}{6}\right) = 10$, what is the value of $s\left(\frac{3\pi}{2}\right)$?

$s\left(\frac{3\pi}{2}\right) = 10 + \int_{\pi/6}^{3\pi/2} \cos t \, dt$

$= 10 + \sin t \Big|_{\pi/6}^{3\pi/2}$

$10 + \sin\left(\frac{3\pi}{2}\right) - \sin\left(\frac{\pi}{6}\right)$

$10 + (-1) - \left(\frac{1}{2}\right)$

$\boxed{8.5 \text{ km}}$

(b) What is the net change in distance over the first π months?

$\int_0^{\pi} \cos t \, dt$

$\sin t \Big|_0^{\pi} = \sin \pi - \sin 0 = 0 - 0 = \boxed{0}$

(c) What is the total distance traveled by the particle during the first π months? Show the set up AND your answer.

$v(t) = 0$

$\cos t = 0$

$t = \frac{\pi}{2}$

$\int_0^{\pi/2} \cos t \, dt + \int_{\pi/2}^{\pi} \cos t \, dt$

$\sin t \Big|_0^{\pi/2} + \sin t \Big|_{\pi/2}^{\pi}$

$1 - 0 + 0 - (-1) = 1 + 1 = \boxed{2}$

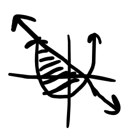
6. Find the area between the two curves $y = x^2 - 4$ and $y = 2 - x$.

$x^2 - 4 = 2 - x$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3 \quad x = 2$



$\int_{-3}^{2} \left[(2-x) - (x^2-4)\right] dx$

$\int_{-3}^{2} (-x^2 - x + 6) \, dx$

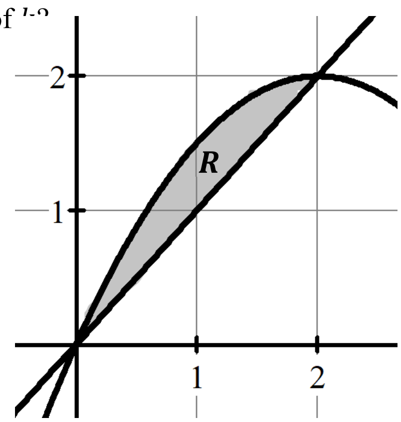
$-\frac{x^3}{3} - \frac{x^2}{2} + 6x \Big|_{-3}^{2}$

$\left(-\frac{8}{3} - \frac{4}{2} + 12\right) - \left(9 - \frac{9}{2} - 18\right)$

$-\frac{8}{3} + 10 + 9 + \frac{9}{2}$

$\approx \boxed{20.833}$

7. **Calculator active.** Let R be the region bounded by the graphs $y = 2x - \frac{1}{2}x^2$ and $y = x$ as sown in the figure. If the line $x = k$ divides R into two regions of equal area, what is the value of k?



$\int_0^k \left(2x - \frac{1}{2}x^2 - x\right) dx = \int_k^2 \left(2x - \frac{1}{2}x^2 - x\right) dx$

$\int_0^k \left(x - \frac{1}{2}x^2\right) dx = \int_k^2 \left(x - \frac{1}{2}x^2\right) dx$

$\left[\frac{x^2}{2} - \frac{x^3}{6}\right] \Big|_0^k = \left[\frac{x^2}{2} - \frac{x^3}{6}\right] \Big|_k^2$

$\frac{k^2}{2} - \frac{k^3}{6} - 0 = 2 - \frac{4}{3} - \left(\frac{k^2}{2} - \frac{k^3}{6}\right)$

$k^2 - \frac{k^3}{3} = 2 - \frac{4}{3}$

$\boxed{k = 1}$

8. **Calculator active.** A 10,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \frac{400t}{t+2} \text{ for } 0 \le t \le 6.$$

a. Find $\int_0^6 r(t)\, dt$

$$\boxed{1290.9645}$$

b. Explain the meaning of your answer to part *a* in the context of this problem.

1,290.964 liters have drained out of the tank during the first six hours.

c. Write, but do not solve, an equation involving an integral to find the time A when the amount of water in the tank is 8,000 liters.

$$\boxed{10{,}000 - \int_0^A \frac{400t}{t+2}\, dt = 8{,}000}$$

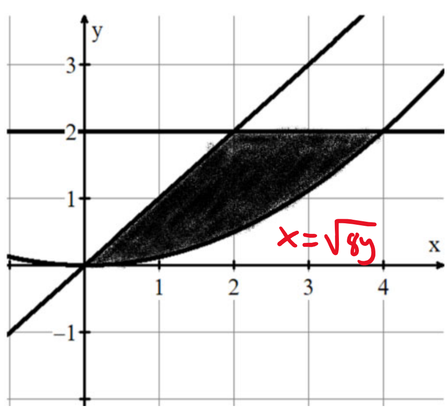
It is "minus" because $r(t)$ represents the rate of water draining from the tank.

Set up the integral(s) that give the area of the region bounded by the given equations. Show the equivalent set up with respect to x as well as with respect to y.

9. $y = x,\ \ y = \frac{x^2}{8},\ \ y = 2$

with respect to x

$$\boxed{\int_0^2 \left(x - \frac{x^2}{8}\right) dx + \int_2^4 \left(2 - \frac{x^2}{8}\right) dx}$$



with respect to y

$x = y \qquad 8y = x^2$

$\pm\sqrt{8y} = x$

$$\boxed{\int_0^2 \left(\sqrt{8y} - y\right) dy}$$

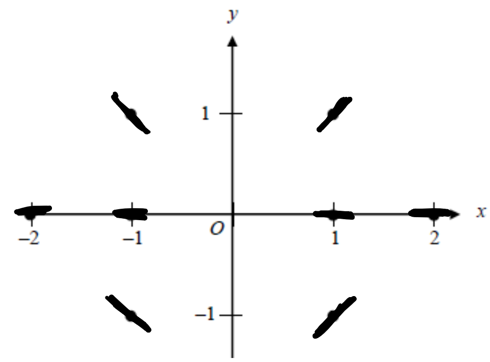
Name: Solutions Date:______________ Period: _____ **Review**

Unit 7 Review – Differential Equations

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 7.

1. Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x}$, where $x \neq 0$.

   a. On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.



   b. Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = -1$.

$y^{-2}dy = \frac{1}{x}dx$

$-\frac{1}{y} = \ln|x| + C$

$1 = C$

$-\frac{1}{y} = \ln|x| + 1$

$$\boxed{y = -\frac{1}{\ln|x|+1}}$$

   c. Write an equation for the tangent line to the curve $y = f(x)$ through the point $(1, -1)$. Then use your tangent line equation to estimate the value of $f(1.2)$.

$\left.\frac{dy}{dx}\right|_{(1,-1)} = \frac{1}{1} = 1$

$y + 1 = 1\cdot(x-1)$

$y + 1 = (1.2 - 1)$

$y + 1 = 0.2$

$$\boxed{y = -0.8}$$

2. The rate of change of the volume, $V(t)$, of water in a swimming pool is directly proportional to the cube root of the volume. If $V = 27$ ft^3 when $\frac{dV}{dt} = 5$, what is a differential equation that models this situation?

$\frac{dV}{dt} = k\sqrt[3]{V}$

$5 = k\sqrt[3]{27}$

$\frac{5}{3} = k$

$$\boxed{\frac{dV}{dt} = \frac{5}{3}\sqrt[3]{V}}$$

Find the general solution of the differential equation.

3. $\frac{dy}{dx} = \frac{2x}{y}$

$y\,dy = 2x\,dx$

$\frac{y^2}{2} = x^2 + C_1$

$y^2 = 2x^2 + C_2$

$$\boxed{y = \pm\sqrt{2x^2 + C}}$$

4. $\frac{dy}{dx} = x(y+4)$

$\frac{1}{y+4}dy = x\,dx$

$\ln|y+4| = \frac{x^2}{2} + C_1$

$y + 4 = e^{\frac{1}{2}x^2 + C_1}$

$$\boxed{y = Ce^{x^2/2} - 4}$$

For each differential equation, find the particular solution that passes through the given point.

5. $\frac{dy}{dx} = \frac{18}{6x+3} + \frac{4}{x^3}$; $\left(-\frac{1}{3}, -15\right)$

$u = 6x+3$

$\frac{du}{6} = dx$

$y = 18 \cdot \frac{1}{6} \ln|6x+3| - \frac{4}{2x^2} + C$

$-15 = 3\ln|-2+3| - \frac{4}{2(\frac{1}{9})} + C$

$-15 = 0 - 4\left(\frac{9}{2}\right) + C$

$-15 = -18 + C$

$3 = C$

$\boxed{y = 3\ln|6x+3| - \frac{2}{x^2} + 3}$

6. $\frac{dy}{dx} = 2y$ and $y = -0.2$ when $x = 0$

$\frac{1}{y} = 2\,dx$

$\ln|y| = 2x + C$

$|y| = Ce^{2x}$

$0.2 = C$

$|y| = 0.2e^{2x}$

$\boxed{y = -0.2e^{2x}}$

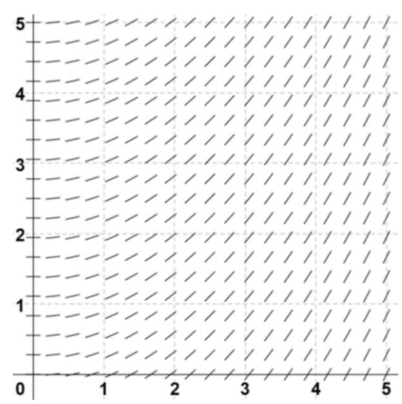
7. A population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 12 years, then what is the value of k?

$2C = Ce^{k(12)}$

$\ln 2 = 12k$

$\boxed{k = \frac{\ln 2}{12}}$

8. Explain why the following slope field cannot represent the differential equation $\frac{dy}{dt} = 0.4y$



One possible answer: When $y = 0, \frac{dy}{dt} = 0$. However, in the slope field, the slopes of the line segments for $y = 0$ are nonzero.

9. For what value of k, if any, will $y = k\cos(2x) + 3\sin(4x)$ be a solution to the differential equation $y'' + 16y = -6\cos(2x)$?

$y' = -2k\sin(2x) + 12\cos(4x)$

$\underbrace{y'' = -4k\cos(2x) - 48\sin(4x)}_{y''} + \underbrace{16\left[k\cos(2x) + 3\sin(4x)\right]}_{16y} = -6\cos(2x)$

$12k\cos(2x) = -6\cos(2x)$

$\boxed{k = -\frac{1}{2}}$

Name: Solutions Date:_______________ Period: ______ **Review**

End-of-Unit 6 Review – Integration and Accumulation of Change

Lessons 6.6 through 6.14

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 6.

Find the value of the definite integral.

1. $\int_{-2}^{-1}\left(\frac{1}{x^2}+x^2-5x\right)dx$

$-\frac{1}{x}+\frac{x^3}{3}-\frac{5x^2}{2}\Big|_{-2}^{-1}$

$\left[-\frac{1}{(-1)}+\frac{-1}{3}-\frac{5}{2}\right]-\left[-\frac{1}{(-2)}-\frac{8}{3}-\frac{20}{2}\right]$

$\left[1-\frac{1}{3}-\frac{5}{2}\right]-\left[\frac{1}{2}-\frac{8}{3}-10\right]$

$11+\frac{7}{3}-\frac{6}{2}$

$8+\frac{7}{3}$

$\frac{24}{3}+\frac{7}{3}=\boxed{\frac{31}{3}}$

2. $\int_{-1}^{8}\left(x^{2/3}-x\right)dx$

$\frac{x^{5/3}}{5/3}-\frac{x^2}{2}\Big|_{-1}^{8}$

$\frac{3}{5}\left(\sqrt[3]{x}\right)^5-\frac{1}{2}x^2\Big|_{-1}^{8}$

$\left[\frac{3}{5}(2)^5-32\right]-\left[-\frac{3}{5}-\frac{1}{2}\right]$

$\left[\frac{96}{5}-32\right]-\left[-\frac{6}{10}-\frac{5}{10}\right]$

$\frac{192}{10}-\frac{320}{10}+\frac{11}{10}$

$\boxed{-\frac{117}{10}}$

3. $\int_0^{\pi}(x-\sin x)\,dx$

$\frac{x^2}{2}+\cos x\Big|_0^{\pi}$

$\left[\frac{\pi^2}{2}+(-1)\right]-\left[0+1\right]$

$\frac{\pi^2}{2}-1-1$

$\boxed{\frac{\pi^2}{2}-2}$

4. $\int_{-1}^{1}x\sqrt{1-x^2}\,dx$

$u=1-x^2$

$\frac{du}{-2x}=dx$

$\int_0^0 x\sqrt{u}\,\frac{du}{-2x}$

$-\frac{1}{2}\int_0^0\sqrt{u}\,du$

$\boxed{0}$

lower band = upper band

5. $\int_0^{\frac{\pi}{6}}\frac{\sin(2x)}{\cos^2(2x)}dx$

$u=\cos(2x)$

$du=-\sin(2x)\cdot 2dx$

$\frac{du}{-2\sin(2x)}=dx$

$\int_1^{1/2}\frac{\sin(2x)}{u^2}\left(\frac{du}{-2\sin(2x)}\right)$

$-\frac{1}{2}\int_1^{1/2}u^{-2}\,du$

$-\frac{1}{2}\left[\frac{u^{-1}}{-1}\right]\Big|_1^{1/2}$

$\frac{1}{2}\left[\frac{1}{1/2}-\frac{1}{1}\right]=\frac{1}{2}[1]=\boxed{\frac{1}{2}}$

6. $\int_e^{e^2}\frac{1}{x\ln x}dx$

$u=\ln x$

$du=\frac{1}{x}dx$

$x\,du=dx$

$\int_1^2\frac{1}{xu}\cdot(x\,du)$

$\int_1^2\frac{1}{u}\,du$

$\ln|u|\Big|_1^2$

$\ln 2-\ln 1$

$\boxed{\ln 2}$

7. If $\int_{-5}^{2}f(x)dx=-17$ and $\int_5^2 f(x)dx=4$, what is the value of $\int_{-5}^{5}f(x)dx$?

$\int_{-5}^{5}f(x)\,dx=\int_{-5}^{2}f(x)\,dx+\int_{2}^{5}f(x)\,dx$

$=(-17)+(-4)$

(A) -21 (B) -13 (C) 0 (D) 13 (E) 21

Find the following indefinite integrals.

8. $\int \left(\frac{x^2-x+5}{x}\right) dx$

$\int \left(x-1+\frac{5}{x}\right) dx$

$\boxed{\frac{x^2}{2} - x + 5\ln x + C}$

9. $\int \sec x \tan x \, dx$

$\boxed{\sec x + C}$

10. $\int (e^x + 2^x)\, dx$

$\boxed{e^x + \frac{1}{\ln 2} 2^x + C}$

11. $\int \left(\frac{1}{x}+\frac{1}{x^3}\right) dx$

$\ln|x| + \frac{x^{-2}}{-2} + C$

$\boxed{\ln|x| - \frac{1}{2x^2} + C}$

12. $\int \sqrt{x}\left(x-\frac{4}{x}\right) dx$

$\int x^{3/2} - 4x^{-1/2} \, dx$

$\frac{x^{5/2}}{5/2} - \frac{4x^{1/2}}{1/2} + C$

$\boxed{\frac{2}{5}x^{5/2} - 8\sqrt{x} + C}$

13. $\int \frac{50x^3-55x^2-26x+33}{10x-7} dx$

$5x^2 - 2x - 4 + \frac{5}{10x-7}$

$10x-7 \overline{\big) 50x^3 - 55x^2 - 26x + 33}$

$-(50x^3 - 35x^2)$

$-20x^2 - 26x + 33$

$-(-20x^2 + 14x)$

$-40x + 33$

$-(-40x + 28)$

5

$\int 5x^2 - 2x - 4 + \frac{5}{\underbrace{10x-7}} \, dx$

$u = 10x - 7$

$\frac{du}{10} = dx$

$\frac{5x^3}{3} - \frac{2x^2}{2} - 4x + \frac{5}{10}\ln|10x-7| + C$

$\boxed{\frac{5}{3}x^3 - x^2 - 4x + \frac{1}{2}\ln|10x-7| + C}$

14. $\int \frac{1}{x^2+2x+2} dx$

$(x^2+2x+1)+2-1$

$\int \frac{1}{(x+1)^2+1} dx$

$\boxed{\tan^{-1}(x+1) + C}$

15. **Calculator active problem.** If $f'(x) = \sin(e^x)$ and $f(0) = 5.7$, then $f(2) =$

$\boxed{5.7 + \int_0^2 \sin(e^x)\, dx \approx 6.2509}$

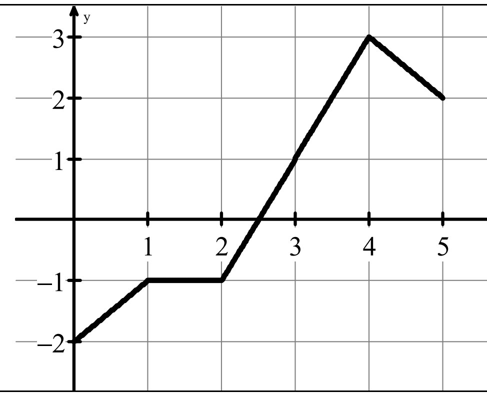
Name: Solutions Date:______________ Period: _____ **Review**

Mid-Unit 6 Review – Integration and Accumulation of Change

Lessons 6.1 through 6.5

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 6.

1. Let $g(x) = \int_a^x f(t)\,dt$ with the graph of f shown above and a is a constant. Find the x-values of g regarding each of the following conditions.

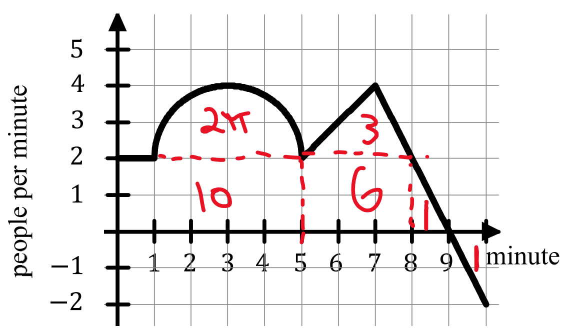


a. Relative minimum(s) $x = 2.5$	b. Relative maximum(s) none	
c. Concave up $(0,1)$ and $(2,4)$	d. Concave down $(4,5)$	
e. Increasing $(2.5, 5)$	f. Decreasing $(0, 2.5)$	g. Point(s) of inflection $x = 4$

h. Given $h(x) = \int_0^{\frac{x}{4}+1} f(t)\,dt$. Find the x-value where h has a relative minimum.

$\frac{x}{4} + 1 = 2.5$ $\quad \frac{x}{4} = 1.5$ $\quad x = 6$

2. The graph below shows the rate of change of the number of people in line for a concert.



a. How many people has the line gained or lost after 5 minutes? Round or truncate to 3 decimal places.

gained $10 + 2\pi$ people

(16.283)

b. How many people has the line gained or lost after 10 minutes? Round or truncate to 3 decimal places.

gained $19 + 2\pi$ People

(25.283)

3. The graph shows which of the following?

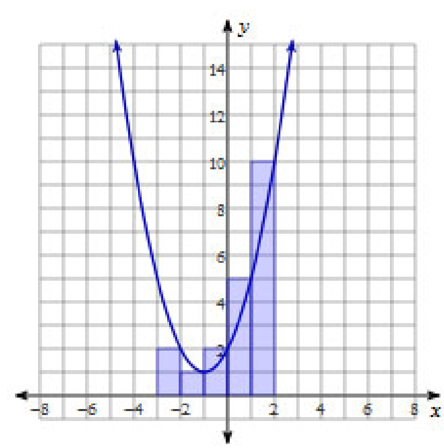
(A) Left Riemann Sum with 5 subintervals

(B) Right Riemann Sum with 5 subintervals

(C) Midpoint Riemann Sum with 5 subintervals

(D) Trapezoidal Approximation with 5 subintervals

(E) None of the above



4. Use a **Left**-Riemann sum with 4 subintervals to approximate the integral based of the values in the table.

$$\int_0^{10} f(x)\,dx$$

x	0	4	6	7	10
$f(x)$	3	2	4	5	7

$4(3) + 2(2) + 1(4) + 3(5)$

$12 + 4 + 4 + 15$

$\boxed{35}$

5. Use a Trapezoidal approximation with 4 subintervals to approximate the area under $f(x) = \frac{1}{4}x^2 - 2x + 6$ on $[-3,0]$

$\text{width} = \frac{0--3}{4} = \frac{3}{4} = 0.75$

$\frac{3}{4}\left(\frac{f(-3)+f(-2.25)}{2}\right) + \frac{3}{4}\left(\frac{f(-2.25)+f(-1.5)}{2}\right) + \frac{3}{4}\left(\frac{f(-1.5)+f(-0.75)}{2}\right) + \frac{3}{4}\left(\frac{f(-0.75)+f(0)}{2}\right)$

$\frac{3}{4}(13.0078125) + \frac{3}{4}(10.6640625) + \frac{3}{4}(8.6015625) + \frac{3}{4}(6.8203125)$

$\boxed{29.320}$

6. Write a definite integral that is equivalent to $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{3}{n}\right)\left(-2 + \frac{3k}{n}\right)^4$. The lower limit for the integral is -2.

$\int_{-2}^{1} x^4\,dx$

Find $F'(x)$.	
7. $F(x) = \int_0^{\cos x} t^2\,dt$	8. $F(x) = \int_{x^2}^{8-x} (2t+5)\,dt$

7.

$\left((\cos x)^2 \cdot (-\sin x)\right)$

$\boxed{-\sin x \cos^2 x}$

8.

$[2(8-x)+5](-1) - [2(x^2)+5](2x)$

$(16-2x+5)(-1) - 4x^3 - 10x$

$-16+2x-5-4x^3-10x$

$\boxed{-4x^3-8x-21}$

Unit 5 Review – Analytical Applications of Differentiation

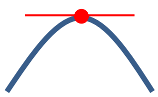
This review summarizes everything from Unit 5 along with examples but contains no problems to work through.

DEFINITIONS

Extrema: The maximum and minimum points. Extrema can be absolute or relative.

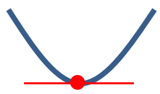
Critical Points: Where the first derivative is zero or DNE. These are possible maximum, minimum, or points of inflection!

$f'(x) = 0$



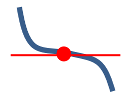
Horizontal Tangent Maximum

$f'(x) = 0$



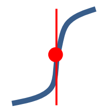
Horizontal Tangent Minimum

$f'(x) = 0$



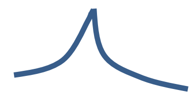
Horizontal Tangent Point of Inflection

$f'(x) = DNE$



Vertical Tangent Point of Inflection

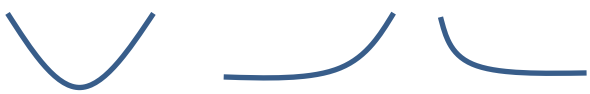
$f'(x) = DNE$



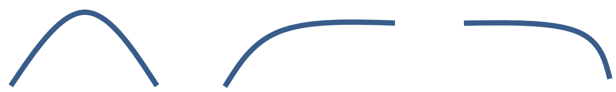
Cusp (No Tangent) Maximum

Concavity: Where the function is "cupping" up or down

CONCAVE UP



CONCAVE DOWN

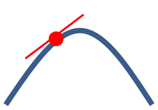


Points of Inflection: Where the second derivative is zero or DNE and changes sign!

FIRST DERIVATIVE

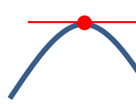
The first derivative is the instantaneous rate of change, or the slope of the tangent line, and can determine if the function is increasing or decreasing at a given point.

$f'(x) > 0$



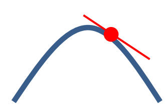
Function is increasing

$f'(x) = 0$



Function is not increasing or decreasing

$f'(x) < 0$

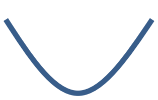


Function is decreasing

SECOND DERIVATIVE

The second derivative determines concavity.

$f''(x) > 0$

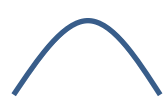


Concave Up

$f''(x) = 0$

Neither concave up or concave down

$f''(x) < 0$



Concave Down

Name: ______________________________ Date:______________ Period: _____ **Review**

Mid-Unit 5 Review – Analytical Applications of Differentiation

Lessons 5.1 through 5.7

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 5.

1. If $y = -2x^2 + 4x + 3$ apply the Mean Value Theorem to find when the instantaneous rate of change will equal the average rate of change on the interval $[1, 3]$.

Avg $\frac{y(3) - y(1)}{3-1} = \frac{-3-5}{2} = -4$

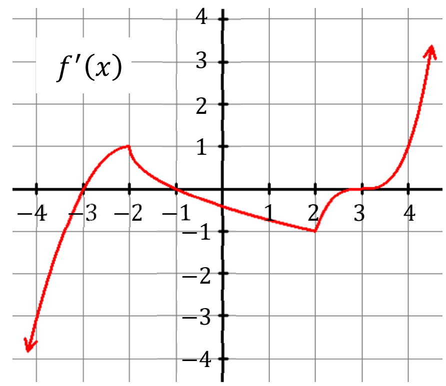
$y' = -4x + 4$

$-4x + 4 = -4$

$-4x = -8$

$\boxed{x = 2}$

2. Below is the graph of f'. Find all relative extrema of f and justify.



Relative minimum at $x = -3$ and $x = 3$ because f' changes sign from negative to positive.

Rel. max at $x = -1$ b/c f' changes sign from pos to neg.

3. The derivative of g is given by $g'(x) = 6x^2 - 6$. Use the First Derivative Test to find all relative extrema and justify your conclusions.

$6x^2 - 6 = 0$

$x^2 = 1$

$x = \pm 1$

x	$(-\infty, -1)$	-1	$(-1, 1)$	1	$(1, \infty)$
$f'(x)$	+	0	−	0	+

Rel. max at $x = -1$ b/c f' changes from pos. to neg.

Rel. min at $x = 1$ b/c f' changes sign from neg. to pos.

4. What is the minimum value of $f(x) = xe^{\frac{x}{3}}$?

$f'(x) = e^{x/3} + \frac{1}{3}xe^{x/3}$

$e^{x/3}\left(1 + \frac{1}{3}x\right) = 0$

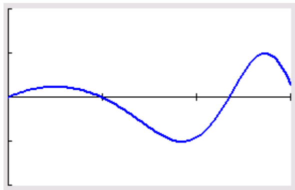
$x = -3$

x	$(-\infty, -3)$	-3	$(3, \infty)$
f'	−	0	+

$f(-3) = -3e^{-1}$

$\boxed{-\frac{3}{e}}$

5. **Calculator active problem.** The derivative of f is defined by $f'(x) = \sin(x - x^2)$ for $0 \le x \le 3$. On what interval(s) is f decreasing?



$$\boxed{1 < x < 2.3416}$$

6. What is the absolute maximum value AND the absolute minimum value of the function $g(x) = x^3 - 12x$ on the closed interval $[0, 4]$.

$g'(x) = 3x^2 - 12$

$3x^2 - 12 = 0$

$x^2 = 4$

$x = \pm 2$

$g(-2)$ is outside the interval

$g(0) = 0$

$g(2) = -16$ ← Abs min

$g(4) = 16$ ← Abs max

7. Use the 2nd Derivative Test to find x-values of the extrema of $g(x) = 2\cos x - x$ on the interval $(0, 2\pi)$ and justify your answer.

$g'(x) = -2\sin x - 1 = 0$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}$ $\quad x = \frac{11\pi}{6}$

$g''(x) = -2\cos x$

$g''\left(\frac{7\pi}{6}\right) = -2\left(-\frac{\sqrt{3}}{2}\right) > 0$

$g''\left(\frac{11\pi}{6}\right) = -2\left(\frac{\sqrt{3}}{2}\right) < 0$

Rel min at $x = \frac{7\pi}{6}$ because $g'\left(\frac{7\pi}{6}\right) = 0$ and $g''\left(\frac{7\pi}{6}\right) > 0$.

Rel. max at $x = \frac{11\pi}{6}$ because $g'\left(\frac{11\pi}{6}\right) = 0$ and $g''\left(\frac{7\pi}{6}\right) < 0$.

8. Find the intervals of concavity for the function $f(x) = x^4 + 4x^3 - 18x^2 - 4x + 7$

$f'(x) = 4x^3 + 12x^2 - 36x - 4$

$f''(x) = 12x^2 + 24x - 36$

$12(x^2 + 2x - 3) = 0$

$12(x+3)(x-1) = 0$

$x = -3 \quad x = 1$ ← Possible pts of inflection

x	$(-\infty, -3)$	-3	$(-3, 1)$	1	$(1, \infty)$
$f''(x)$	+	0	−	0	+

f is concave up on $(-\infty, -3)$ and $(1, \infty)$ because $f''(x) > 0$.

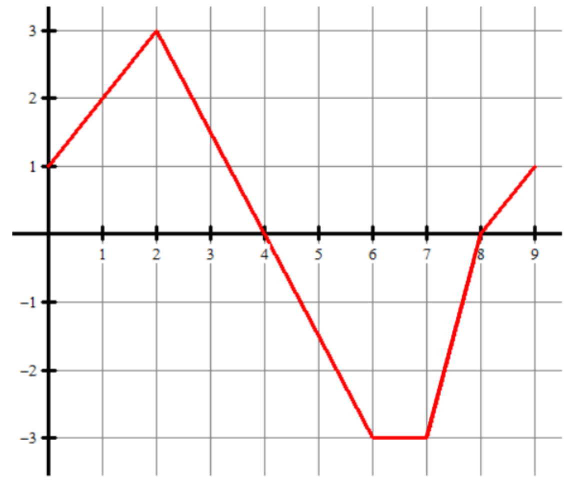
f is concave down on $(-3, 1)$ because $f''(x) < 0$.

Name: Solutions Date:______________ Period: _____ **Review**

Unit 4 REVIEW – Contextual Application of Differentiation

Reviews do NOT cover all material from the lessons but should remind you of key points. To be prepared, you should review all packets from Unit 4.

1. The figure shows the velocity $v = \frac{ds}{dt} = f(t)$ of a body moving along a coordinate line in meters per second.



a) When does the body reverse direction?

$t=4$ $t=8$

b) When is the body moving at a constant speed?

$(6,7)$

c) What is the body's maximum speed?

3 meters per second

d) At what time interval(s) is the body slowing down?

$(2,4)$ and $(7,8)$

Find the following. Use L'Hospital's when possible.

2. $\lim_{x\to 2} \frac{x-2}{x^2-7x+10} = \frac{0}{0}$ $\lim_{x\to 2} \frac{1}{2x-7}$ $\frac{1}{4-7}$ $\boxed{-\frac{1}{3}}$	3. $\lim_{x\to 0} \frac{3x^2}{e^x-1-x} = \frac{0}{0}$ $\lim_{x\to 0} \frac{6x}{e^x-1} = \frac{0}{0}$ $\lim_{x\to 0} \frac{6}{e^x}$ $\boxed{6}$	4. $\frac{d}{dx} \frac{3x-2}{5x+1}$ Quotient Rule! $\frac{(3)(5x+1)-(3x-2)(5)}{(5x+1)^2}$ $15x+3-15x+10$ $\boxed{\frac{13}{(5x+1)^2}}$

5. If the length l of a rectangle is decreasing at a rate of 2 inches per minute while its width w is increasing at a rate of 2 inches per minute, which of the following must be true about the area A of the rectangle?

$\frac{dL}{dt} = -2$

$\frac{dw}{dt} = 2$

$A = LW$

$\frac{dA}{dt} = \frac{dL}{dt}w + L\frac{dw}{dt}$

$\frac{dA}{dt} = -2w + 2L$

$\frac{dA}{dt} = 2(L-w)$

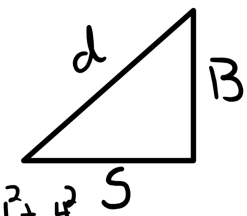
If $L > w$ then $\frac{dA}{dt}$ is positive

(A) A is always increasing. (B) A is always decreasing. (C) A is increasing only when $l > w$.

(D) A is increasing only when $l < w$. (E) A remains constant.

The following problems are calculator active.

6. Brust is riding his bicycle north away from an intersection at a rate of 15 miles per hour. Sully is driving his car towards the intersection from the west at a rate of 30 miles per hour. If Brust is 0.4 miles from the intersection, and Sully is 1 mile from the intersection, at what rate is the distance between the two of them increasing or decreasing?



$d^2 = 1^2 + .4^2$

$d = \sqrt{1.16}$

$\frac{dB}{dt} = 15$

$\frac{ds}{dt} = -30$

$B = 0.4$

$S = 1$

$\frac{dd}{dt} = ?$

$S^2 + B^2 = d^2$

$2S\frac{ds}{dt} + 2B\frac{dB}{dt} = 2d\frac{dd}{dt}$

$-60 + 12 = 2\sqrt{1.16}\,\frac{dd}{dt}$

$\boxed{\frac{dd}{dt} = -22.283 \text{ mph}}$

7. The side of a cube is increasing at a constant rate of 0.2 centimeters per second. In terms of the surface area A, what is the rate of change of the volume of the cube, in cubic centimeters per second?

$V = s^3$

$A = 6s^2$

$\frac{ds}{dt} = 0.2$

$\frac{dv}{dt} = 3s^2\frac{ds}{dt}$

$\frac{dv}{dt} = 3s^2(0.2)$ → $\frac{dv}{dt} = 0.6s^2$

★ If we multiply by 10, we get A. or $\frac{A}{10} = \frac{dv}{dt}$

(A) $0.1A$ (B) $0.2A$ (C) $0.6A$

(D) $0.04A$ (E) $0.008A$

8. The function $f(x) = (1 - \sin x)^2$ is concave up at $x = \frac{\pi}{6}$?

a. What is the estimate for $f(0.5)$ using the local linear approximation for f at $x = \frac{\pi}{6}$?

$f'(x) = 2(1 - \sin x)(-\cos x)$

$f'(\frac{\pi}{6}) = 2(1 - \frac{1}{2})(-\frac{\sqrt{3}}{2})$

$m = -\frac{\sqrt{3}}{2}$

$y_1 = f(\frac{\pi}{6}) = (1 - \frac{1}{2})^2 = \frac{1}{4}$

$y - \frac{1}{4} = -\frac{\sqrt{3}}{2}(x - \frac{\pi}{6})$

$y - \frac{1}{4} = -\frac{\sqrt{3}}{2}(0.5 - \frac{\pi}{6})$

$\boxed{f(0.5) \approx 0.270}$

b. Is it an underestimate or overestimate? Explain.

Underestimate because f(x) is concave up.

Name: Solutions Date:________________ Period: ______

Review

Unit 3 REVIEW – Composite, Implicit, and Inverse Functions

Reviews do NOT cover all material from the lessons but should remind you of key points. To be prepared, you should review all packets from Unit 3.

Find the derivative.

1. $h(x) = \cos^2(4x)$

$$h'(x) = 2\cos(4x) \cdot (-\sin(4x)) \cdot 4$$

$$\boxed{h'(x) = -8\cos(4x)\sin(4x)}$$

2. $y = \ln\sqrt{x+3}$

$$y' = \frac{1}{\sqrt{x+3}} \cdot \frac{1}{2\sqrt{x+3}}$$

$$\boxed{y' = \frac{1}{2x+6}}$$

3. $x^2 + 2y^5 = 10xy$

$$2x + 10y^4\frac{dy}{dx} = 10y + 10x\frac{dy}{dx}$$

$$\frac{dy}{dx}(10y^4 - 10x) = 10y - 2x$$

$$\boxed{\frac{dy}{dx} = \frac{5y - x}{5y^4 - 5x}}$$

4. $y = \csc^{-1}(x^3)$

$$\frac{dy}{dx} = -\frac{1}{|x^3|\sqrt{x^6 - 1}} \cdot (3x^2)$$

$$\boxed{\frac{dy}{dx} = -\frac{3}{|x|\sqrt{x^6 - 1}}}$$

For each problem, let f and g be differentiable functions where $g(x) = f^{-1}(x)$ for all x.

5. $f(6) = -1$, $f(4) = -2$, $f'(6) = 3$, and $f'(4) = 7$. What is the value of $g'(-1)$?

$$g'(-1) = \frac{d}{dx} f^{-1}(-1) = \frac{1}{f'(f^{-1}(-1))}$$

$$= \frac{1}{f'(6)}$$

$$= \boxed{1/3}$$

6. Let f be the function defined by $f(x) = x^3 + 3x + 1$. Let $g(x) = f^{-1}(x)$, where $g(-3) = -1$. What is the value of $g'(-3)$?

$$g'(-3) = \frac{d}{dx} f^{-1}(-3) = \frac{1}{f'(f^{-1}(-3))}$$

$$f'(x) = 3x^2 + 3$$

$$f'(-1) = 6$$

$$= \frac{1}{f'(-1)}$$

$$= \boxed{1/6}$$

Find $\frac{d^2y}{dx^2}$ based on the given information.

7. $y = x^5 - e^{4x}$

$$\frac{dy}{dx} = 5x^4 - e^{4x} \cdot 4$$

$$\frac{d^2y}{dx^2} = 20x^3 - e^{4x} \cdot 4 \cdot 4$$

$$\boxed{\frac{d^2y}{dx^2} = 20x^3 - 16e^{4x}}$$

8. $y = y^2 + x$

$$\frac{dy}{dx} = 2y\frac{dy}{dx} + 1$$

$$\frac{dy}{dx}(1-2y) = 1$$

$$\frac{dy}{dx} = \frac{1}{(1-2y)} = (1-2y)^{-1}$$

$$\frac{d^2y}{dx^2} = -1(1-2y)^{-2} \cdot \left(-2\frac{dy}{dx}\right)$$

$$= -\frac{1}{(1-2y)^2} \cdot \left(\frac{-2}{(1-2y)}\right) = \boxed{\frac{2}{(1-2y)^3}}$$

9. Find the equation of the tangent line.
$x^2 + 7y^2 = 8y^3$ at $(-6, 2)$

$$2x + 14y\frac{dy}{dx} = 24y^2\frac{dy}{dx}$$

$$2(-6) + 14(2)\frac{dy}{dx} = 24(4)\frac{dy}{dx}$$

$$-12 + 28\frac{dy}{dx} = 96\frac{dy}{dx}$$

$$-12 = 68\frac{dy}{dx}$$

$$-\frac{3}{17} = \frac{dy}{dx}$$

$$\boxed{y - 2 = -\frac{3}{17}(x+6)}$$

10. If $x = y^2 - \cos x$ find $\frac{d^2y}{dx^2}$ at $(0, -1)$.

$$1 = 2y\frac{dy}{dx} + \sin x$$

$$\frac{1-\sin x}{2y} = \frac{dy}{dx} \qquad \left.\frac{dy}{dx}\right|_{(0,-1)} = \frac{1-\sin(0)}{2(-1)} = \frac{1}{-2}$$

$$\frac{d^2y}{dx^2} = \frac{-\cos x\,(2y) - (1-\sin x)\left(2\frac{dy}{dx}\right)}{4y^2} \qquad = -\tfrac{1}{2}$$

$$\left.\frac{d^2y}{dx^2}\right|_{(0,-1)} = \frac{-(1)(-2) - (1-0)(-1)}{4(-1)^2} = \frac{2+1}{4} = \boxed{\frac{3}{4}}$$

Name: Solutions Date:________ Period: ____ **Review**

Unit 2 Review – Differentiation: Definition & Fundamental Properties

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you should review all packets from Unit 2.

Find the average rate of change of each function on the given interval. Use appropriate units if necessary.

1. $w(x) = \ln x; \quad 1 \le x \le 7$

$$\frac{W(7)-W(1)}{7-1} = \frac{\ln 7 - \ln 1}{6} = \boxed{\frac{\ln 7}{6}}$$

2. $s(t) = -t^2 - t + 4; \; [1,5]$
t represents seconds
s represents feet

$$\frac{S(5)-S(1)}{5-1} = \frac{-26-2}{4} = \boxed{-7 \text{ ft/sec}}$$

3. Find the derivative of $y = 2x^2 + 3x - 1$ by using the definition of the derivative. $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$\frac{dy}{dx} = \lim_{h\to 0}\frac{2(x+h)^2+3(x+h)-1-[2x^2+3x-1]}{h}$$

$$\lim_{h\to 0}\frac{2(x^2+2hx+h^2)+3x+3h-1-2x^2-3x+1}{h}$$

$$\lim_{h\to 0}\frac{2x^2+4hx+2h^2+3h-2x^2}{h} = \lim_{h\to 0}\frac{h(4x+2h+3)}{h} = \boxed{4x+3}$$

4. For the function $h(t)$, h is the temperature of the oven in Fahrenheit, and t is the time measured in minutes.
   a. Explain the meaning of the equation $h(15) = 420$.

   The oven is 420 degrees Fahrenheit after 15 minutes.

   b. Explain the meaning of the equation $h'(43) = -11$.

   The oven is getting cooler by 11 degrees per minute on the 43rd minute.

Find the derivative of each function.

5. $f(x) = 4 - \frac{1}{2x^2}$

$f(x) = 4 - \frac{1}{2}x^{-2}$

$f'(x) = x^{-3}$

$\boxed{f'(x) = \frac{1}{x^3}}$

6. $g(x) = 3\sqrt{x} - \frac{6}{x^2} + 5\pi^3$

$g(x) = 3x^{1/2} - 6x^{-2} + 5\pi^3$

$g'(x) = \frac{3}{2}x^{-1/2} + 12x^{-3}$

$\boxed{g'(x) = \frac{3}{2\sqrt{x}} + \frac{12}{x^3}}$

7. $h(x) = 4e^x - 2\cos x$

$\boxed{h'(x) = 4e^x + 2\sin x}$

8. $s(t) = t^2\sin(t)$

$\boxed{s'(t) = 2t\sin(t) + t^2\cos(t)}$

9. $d(t) = 3\sqrt{t}\ln t$

$d'(t) = \frac{3}{2}t^{-1/2}\ln(t) + 3\sqrt{t}\,\frac{1}{t}$

$\boxed{d'(t) = \frac{3\ln(t)}{2\sqrt{t}} + \frac{3\sqrt{t}}{t}}$

10. $y = \frac{4}{x} - \sec x$

$y = 4x^{-1} - \sec x$

$\boxed{\frac{dy}{dx} = -\frac{4}{x^2} - \sec(x)\tan(x)}$

11. $h(x) = \frac{2-x}{x+2}$

$h'(x) = \frac{(-1)(x+2) - (2-x)(1)}{(x+2)^2} = \frac{-x-2-2+x}{(x+2)^2}$

$\boxed{h'(x) = \frac{-4}{(x+2)^2}}$

Find the equation of the tangent line of the function at the given *x*-value.

12. $f(x) = -2x^3 + 3x$ at $x = -1$.

$y - y_1 = m(x - x_1)$

$f(-1) = -2(-1) + 3(-1) = -1 \leftarrow y_1$

$f'(x) = -6x^2 + 3$

$f'(-1) = -6(1) + 3 = -3 \leftarrow m$

$\boxed{y + 1 = -3(x+1)}$

13. $f(x) = 4\sin x - 2$ at $x = \pi$

$f(\pi) = 4\sin(\pi) - 2 = -2$

$f'(x) = 4\cos x$

$f'(\pi) = 4\cos(\pi) = 4(-1) = -4$

$\boxed{y + 2 = -4(x - \pi)}$

14. Find the equation for the normal line of $y = \frac{1}{2}x^2 + \frac{3}{4}x - 4$ at $x = -3$

$y(-3) = \frac{1}{2}(9) + \frac{3}{4}(-3) - 4$

$y(-3) = \frac{9}{2} - \frac{9}{4} - 4$ y_1

$y(-3) = \frac{18}{4} - \frac{9}{4} - \frac{16}{4} = -\frac{7}{4}$

$y' = x + \frac{3}{4}$

$y'(-3) = -3 + \frac{3}{4} = -\frac{12}{4} + \frac{3}{4} = -\frac{9}{4}$ m

$\boxed{y + \frac{7}{4} = \frac{4}{9}(x+3)}$

15. If $f(x) = 3\sin x - 2e^x$ find $f'(0)$. No calculator!

$f'(x) = 3\cos x - 2e^x$

$f'(0) = 3\cos 0 - 2e^0 = 3 - 2 = \boxed{1}$

A calculator is allowed on the following problems.

16. If $f(x) = x\sin(3x^2 - 2)$; find $f'(7)$.

$\boxed{f'(7) = 260.246}$

Math 8

17. If $f(x) = \csc(3x)$ at $x = 2$.

$\boxed{f'(2) = -36.899}$

18. Use the table below to estimate the value of $d'(120)$. Indicate units of measures.

t seconds	2	13	60	180	500
$d(t)$ feet	10	81	412	808	2,105

$\frac{808 - 412}{180 - 60} = \boxed{3.3 \text{ ft/sec}}$

19. Is the function differentiable at $x = 2$?

$$f(x) = \begin{cases} 3x - 3x^2 - 5, & x < 2 \\ 7 - 9x, & x \geq 2 \end{cases}$$

$3(2) - 3(2)^2 - 5 = 7 - 9(2)$

$6 - 12 - 5 = 7 - 18$

✓ $-11 = -11$

continuous

$3 - 6(2) = -9$

$3 - 12 = -9$

$-9 = -9$ ✓

$\boxed{\text{Yes!}}$

---

20. What values of a and b would make the function differentiable at $x = 4$?

$$f(x) = \begin{cases} a\sqrt{x} + bx^2 - 1, & x < 4 \\ \dfrac{16}{x} + bx, & x \geq 4 \end{cases}$$

$a(\sqrt{4}) + b(16) - 1 = \frac{16}{4} + b(4)$

$2a + 16b - 1 = 4 + 4b$

$2\underline{a} = 5 - 12b$

$2(-4 - 28b) = 5 - 12b$

$-8 - 56b = 5 - 12b$

$-44b = 13$

$\boxed{b = -\frac{13}{44}}$

$\frac{a}{2\sqrt{4}} + 2b(4) = -\frac{16}{(4)^2} + b$

$\frac{a}{4} + 8b = -1 + b$

$\frac{a}{4} = -1 - 7b$

$a = \underline{-4 - 28b}$

$a = -4 - 28\left(-\frac{13}{44}\right)$

$a = -4 + \frac{91}{11}$

$\boxed{a = \frac{47}{11}}$

Name: Answer Key Date:______________ Period: _____ **Review**

End of Unit 1 Review– Limits and Continuity

Lessons 1.10 through 1.16.

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you should review all packets from Unit 1 (including the Mid-Unit Review).

1. If $f(x) = \frac{x+3}{x^2-2x-15}$, identify the type of each discontinuity and where it is located.

$(x+3)(x-5)$

$\boxed{\text{hole at } x=-3 \quad \text{V.A. at } x=5}$

State whether the function is continuous at the given *x* values. Justify your answers!

2. $f(x) = \begin{cases} \cos(3x), & x < 0 \\ \tan x, & 0 \le x < \frac{\pi}{4} \\ \sin(2x), & x \ge \frac{\pi}{4} \end{cases}$

Continuous at $x = 0$?

No because $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

Continuous at $x = \frac{\pi}{4}$?

Yes. $f(\frac{\pi}{4}) = 1$ and $\lim_{x\to \frac{\pi}{4}} f(x) = f(\frac{\pi}{4})$

$\cos(0) = 1$
$\tan(0) = 0$
$\tan(\frac{\pi}{4}) = 1$
$\sin(\frac{\pi}{2}) = 1$

Find the domain of each function.

3. $h(t) = \frac{\sqrt{t+3}}{t-5}$

$\boxed{t \ge -3 \quad t \neq 5}$

4. $f(x) = \ln\left(\frac{2}{x-1}\right)$

$x - 1 \neq 0$

$x \neq 1$

$\frac{2}{x-1} > 0$

therefore $x-1 > 0$

$x > 1$

$\boxed{x > 1}$

5. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2+6x+8}{x+2}$ when $x \neq -2$, then $f(-2) =$

$\frac{\cancel{(x+2)}(x+4)}{\cancel{x+2}}$

$-2+4 = \boxed{2}$

6. Let f be the function defined by $f(x) = \begin{cases} \frac{x^2+8x+12}{x+6}, & x \neq -6 \\ b, & x = -6 \end{cases}$. For what value of b is f continuous at $x = -6$?

$\frac{\cancel{(x+6)}(x+2)}{\cancel{x+6}}$

$x+2 = b$

$(-6)+2 = b$

$\boxed{-4 = b}$

Evaluate the limit.		
7. $\lim_{x\to\infty} \sin\left(\frac{x+3\pi x^2}{2x^2}\right)$ $\sin\left(\frac{3\pi}{2}\right)$ $\boxed{-1}$	8. $\lim_{x\to-5^-} \frac{-3}{25-x^2}$ $\frac{-3}{25-(-5.0001)^2}$ $\frac{-3}{-0.00001}$ $\boxed{\infty}$	9. $\lim_{x\to\infty} \frac{\sin x}{x}$ $\boxed{0}$
10. $\lim_{x\to\infty} \frac{4x^5-2x^2+3}{3x^2+2x^5-x^4}$ $\frac{4}{2}$ $\boxed{2}$	11. $\lim_{x\to-1} \frac{x^2+1}{x+1}$ $\frac{(-1.001)^2+1}{-1.001+1}$ $\frac{(-0.999)^2+1}{-0.999+1}$ $\frac{2.00001}{-0.0001}$ $\frac{1.99999}{0.0001}$ $-\infty$ $\boxed{\text{DNE}}$ ∞	12. $\lim_{x\to\infty} x^5 3^{-x}$ $\frac{x^5}{3^x}$ $\boxed{0}$

13. Identify all horizontal asymptotes of $f(x) = \frac{\sqrt{16x^6+x^3+5x}}{5x^3-8x}$.

as $x \to \infty$ $\boxed{y = 4/5}$

as $x \to -\infty$ $\boxed{y = -4/5}$

Name: Solutions Date:________________ Period: ______ **Review**

Mid-Unit 1 Review – Limits

Lessons 1.1 through 1.9

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 1.

A salesman tracks the number of cars he sells through the model c, where $c(m)$ is number of cars sold and m is the month for $0 \le m \le 24$.		
1. What does $c(10)$ represent? The number of cars sold in the 10th month.	2. What does $\frac{c(16)-c(8)}{16-8}$ represent? The average rate of change of the number of cars sold between the 8th and 16th months.	3. What does $\frac{c(7)-c(6.999)}{7-6.999}$ represent? An estimate of the rate of change of cars being sold (per month) on the 7th month.

Evaluate the limit.

4. $\lim_{x\to 0} \frac{\sqrt{x+19}-\sqrt{19}}{x} \cdot \frac{\sqrt{x+19}+\sqrt{19}}{\sqrt{x+19}+\sqrt{19}}$

$$\frac{(x+19)-(19)}{x(\sqrt{x+19}+\sqrt{19})}$$

$$\frac{x}{x(\sqrt{x+19}+\sqrt{19})}$$

$$\frac{1}{\sqrt{x+19}+\sqrt{19}}$$

$$\boxed{\frac{1}{2\sqrt{19}}}$$

5. $\lim_{x\to -3} \frac{x+3}{x^2+2x-3} = \frac{x+3}{(x+3)(x-1)}$

$$\lim_{x\to -3} \frac{1}{x-1}$$

$$\boxed{-\frac{1}{4}}$$

6. $\lim_{x\to 0} \frac{\sin^2(3x)}{\sin^2(5x)}$

$$\lim_{x\to 0} \frac{\sin(3x)}{1} \cdot \frac{\sin(3x)}{1} \cdot \frac{1}{\sin(5x)} \cdot \frac{1}{\sin(5x)}$$

$$\frac{3x\sin(3x)}{3x} \cdot \frac{3x\sin(3x)}{3x} \cdot \frac{5x}{5x\sin(5x)} \cdot \frac{5x}{5x\sin(5x)}$$

$$3x \cdot 3x \cdot \frac{1}{5x} \cdot \frac{1}{5x}$$

$$\frac{3\cdot 3}{5\cdot 5}$$

$$\boxed{\frac{9}{25}}$$

7. $\lim_{x\to 2^-} \frac{|x-2|}{x-2}$

$$\frac{|1.999-2|}{1.999-2}$$

$$\frac{0.001}{-0.001}$$

$$\boxed{-1}$$

8. $\lim_{x\to 10} \frac{x^2-5x-50}{x-10} = \frac{(x-10)(x+5)}{x-10}$

$$\lim_{x\to 10} x+5$$

$$\boxed{15}$$

9. $\lim_{x\to 0} \frac{\frac{1}{x+1}-1}{x}$

$$\lim_{x\to 0} \frac{\frac{1}{x+1}-\frac{x+1}{x+1}}{x}$$

$$\lim_{x\to 0} \frac{-x}{x+1} \cdot \frac{1}{x}$$

$$\lim_{x\to 0} \frac{-1}{x+1}$$

$$\boxed{-1}$$

10. If $f(x) = \begin{cases} \sin x, & x < -\pi \\ \tan x & -\pi < x < \frac{\pi}{4} \\ \cos x, & x \geq \frac{\pi}{4} \end{cases}$, find the following:

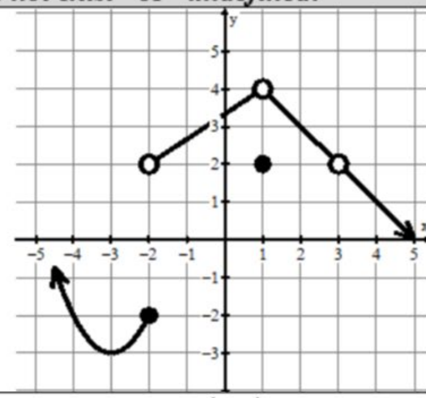
a. $\lim_{x \to -\pi^-} f(x) = 0$

b. $\lim_{x \to -\pi} f(x) = 0$

c. $\lim_{x \to \frac{\pi}{4}} f(x) =$ DNE

d. $f\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

Give the value of each statement. If the value does not exist, write "*does not exist*" or "*undefined.*"



11. $\lim_{x \to 3} f(x) = 2$

12. $\lim_{x \to 1} f(x) = 4$

13. $f(3) =$ DNE

14. $f(-2) = -2$

15. $\lim_{x \to 2} f(x) = 3$

16. $\lim_{x \to -2^+} f(x) = 2$

17. $f(1) = 2$

18. $\lim_{x \to -2^-} f(x) = -2$

19. Let g and h be the functions defined by $g(x) = -\frac{1}{4}x^2 - \frac{1}{2}x - \frac{9}{4}$ and $h(x) = \sin\left(\frac{\pi}{2}x\right) - 1$. If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for all x, what is $\lim_{x \to -1} f(x)$?

$$-\frac{1}{4}(-1)^2 - \frac{1}{2}(-1) - \frac{9}{4} \leq \lim_{x \to -1} f(x) \leq \sin\left(-\frac{\pi}{2}\right) - 1$$

$$-2 \leq \lim_{x \to -1} f(x) \leq -2$$

$$\boxed{-2}$$

CALCULATOR ALLOWED:

20. If $f(x) = \frac{x^2+10x+21}{x+3}$, create your own table of values to help you evaluate $\lim_{x \to -3} f(x)$.

$\lim_{x \to -3} f(x) = 4$

x	-3.1	-3.01	-3.001	-2.999	-2.99	-2.9
$f(x)$	3.9	3.99	3.999	4.001	4.01	4.1